Should monetary policy care about redistribution? Optimal monetary and fiscal policy with heterogeneous agents

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Abstract

We derive optimal monetary policy in a heterogeneous-agent economy with nominal frictions and aggregate shocks. We analyze the model with either sticky prices or sticky wages, and different assumptions about fiscal policy. In the sticky-price economy, when fiscal policy is optimally set, optimal monetary policy implements price stability. Inflation volatility remains low when fiscal policy follows empirically relevant rules, except when the slope of the Phillips curve is large and when the distribution of profits is tilted toward high-productive agents. In the sticky-wage economy, optimal price inflation is much more volatile, but wage inflation stays very small. Under both assumptions regarding rigidity, the lower the productivity of agents, the more they benefit from optimal monetary policy.

Keywords: Heterogeneous agents, optimal Ramsey policies, monetary policy, fiscal policy.

JEL codes: D31, E52, D52, E21.

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1 Introduction

Monetary policy generates redistributive effects through various channels, which have been studied by a vast empirical and theoretical literature. However, it is not clear how these channels should change the conduct of monetary policy. It might be possible that monetary policy should take these effects into account in order to improve welfare, and thus serve a role that is usually played by fiscal policy. Or, on the contrary, perhaps monetary policy should focus solely on monetary goals, and let fiscal tools either dampen or strengthen its redistributive effects. To distinguish between these two claims, one must consider optimal monetary policy in a realistic fiscal environment, where heterogeneity among agents generates a concern for redistribution.

To do so, we follow the so-called Bewley (1980) literature and assume incomplete insurance markets for idiosyncratic risks to be the main source of heterogeneity across agents. This framework is known to be general enough to generate realistic distributions of income and wealth. In this environment, we solve for optimal monetary policy with a realistic fiscal system, considering two forms of nominal frictions: either costly price adjustments or costly nominal wage adjustments. On the one hand, sticky prices have become a standard nominal friction in the heterogeneous-agent literature since the seminal paper of Kaplan et al. (2018), which pioneered the so-called Heterogeneous-Agent New-Keynesian (HANK) models. On the other hand, the slow adjustment of nominal wages is well-documented, and sticky wages may contribute to resolving some issues related to the cyclicality of markups (Nekarda and Ramey, 2020). Indeed, as noted by Tobias et al. (2020) among others, the redistribution of profits is a key force affecting the dynamics of HANK models. As such, we contrast the implications of the two types of nominal rigidities and show that optimal monetary policies markedly differ in the two cases.

In the sticky-price economy, we first assume that fiscal policy is optimally designed, and we jointly solve for optimal monetary and fiscal policies with commitment when five fiscal instruments are available: two linear taxes on real and nominal capital income, a linear tax on labor income, lump-sum transfers, and a riskless one-period public debt. We prove that optimal monetary policy in this framework solely implements price stability and that optimal inflation has no redistributive role over the business cycle. This result can be understood using the distinction made by Kaplan et al. (2018) and Auclert (2019) between the direct and indirect channels of monetary policy. Capital taxes are sufficient instruments for the planner to reproduce any allocation that can be reached with the direct channels, because it affects the returns on capital. The labor tax is sufficient to replicate any allocation reachable with the indirect effects, because it creates a wedge between labor income and the marginal productivity of labor. Thus, linear taxes on capital and labor ensure that monetary policy should focus solely on price stability, while fiscal policy alone deals with redistribution. This equivalence result generalizes the representative-agent result of Correia et al. (2008) to heterogeneous-agent economies. We then relax the assumption of optimal fiscal policy and consider exogenous fiscal rules. In our benchmark calibration, which corresponds to the empirically relevant case of constant marginal tax rates, the planner does not implement perfect price stability, but optimal inflation volatility remains low. Compared to the literature, our analysis confirms both the key roles of assumptions about profit distribution and of the slope of the Phillips curve in generating a sizable inflation response. Inflation volatility is high if the price Phillips curve is steep and, at the same time, profits are both counter-cyclical and disproportionally benefit rich agents.

In the sticky-wage economy, we model the nominal rigidity following Erceg et al. (2000), which is frequently used in the heterogeneous-agent literature (Hagedorn et al., 2019, Alves and Violante, 2023 or Auclert et al., 2024 among others). In this approach, agents are in general not on their labor supply curve, which generates an additional labor market friction. We characterize the optimal policy in the sticky-wage model with the same empirically relevant fiscal system. We find that optimal policy generates a small deviation from nominal wage stability but causes the price level to move sizably, allowing the real wage to adjust to economic shocks. The sticky-wage economy generates a much higher volatility of price inflation (and a much lower volatility of wage inflation) than the sticky-price economy. Consequently, the welfare benefits associated with deviating from price stability are much higher in the sticky-wage economy than in the sticky-price one. Deviation from nominal wage stability is also welfare enhancing, but to a lesser extent. However, we find that under both assumptions regarding rigidity, deviating from nominal stability benefits lower productivity agents far more than higher productivity agents, making it a progressive policy.

Deriving theoretical and quantitative results for optimal policies in incomplete-market economies with aggregate shocks is a challenging task. We perform our analysis thanks to two recent methodological contributions. First, we elaborate on the Lagrangian approach of Marcet and Marimon (2019), which appears particularly well-suited for HANK economies. We introduce the notion of the *net social value of liquidity* (SVL) for each agent, which is the gain obtained when the planner transfers one unit of resources to a particular agent at a given date, considering all general equilibrium effects. The expressions of the SVL differ between the sticky-price case and the sticky-wage case, as the SVL captures different general equilibrium effects. In both cases however, introducing the SVL considerably simplifies the derivation of optimal policies when agents are heterogeneous. Second, to simulate the model, we use a truncated representation of incomplete insurance market economies that we apply here to a monetary economy. This theory of uniform truncation was first used in LeGrand and Ragot (2022a) to study optimal unemployment benefits. We construct a consistent and accurate approximation of the economy in which heterogeneity depends only on a finite but arbitrarily large number of past consecutive realizations of idiosyncratic risk. We also use the refinement of this truncation method, developed in LeGrand and Ragot (2022b), which considers possibly different truncation lengths and solves the curse of dimensionality of the uniform truncation. Both methods provide quantitatively similar results and we check their accuracy by comparing their outcomes to those of the standard

Reiter (2009) method, which is known to be close to other methods such as Boppart et al. (2018) or Auclert et al. (2021). We also consider empirically relevant social welfare functions, which allow us to derive the dynamics of the model around a steady state featuring an empirically relevant fiscal system. This analysis may be of independent interest. Finally and following the literature, we always study the optimal inflation volatility when the economy is close to its long-run equilibrium, which is called the *timeless* perspective (see Woodford, 1999, for instance). As written by McCallum and Nelson (2000), this timeless perspective is the closest notion to "optimal policy making according to a rule". This strategy is also immune to the difficulties arising from the normative analysis of large inflation surprises, which are called period-0 policies in the literature referred below.

Related literature. Our paper belongs to a growing literature that departs from the representative-agent model to analyse optimal monetary policy in heterogeneous-agent models. This earlier literature studied sticky prices or sticky wages (see Woodford, 2003 or Galí, 2015 for textbook treatments and references).

Our equivalence result in a HANK model with sticky prices is related to the five transmission channels of monetary policy that have been identified in the heterogeneous-agent literature (Kaplan et al. 2018, and Auclert, 2019, among others). Monetary policy has direct effects, which operate through changes in real returns (Gornemann et al., 2016). The changes in returns generate a substitution effect, already present in the representative-agent new-Keynesian model (Woodford, 2003). Inflation also affects the real value of nominal assets, through a Fisher effect (Doepke and Schneider, 2006). Changes in real returns generate a wealth effect due to unhedged interest rate exposure, identified by Auclert (2019). In addition to these three direct channels, there are also two indirect effects due to the endogeneity of labor income and to the heterogeneous exposure to income variations (Coibion et al., 2017; Acharya and Dogra, 2021). If labor and capital taxes are optimally designed, then the use of these channels by monetary policy does not increase welfare. Our equivalence results have a similar flavor to that of Correia et al. (2008), who derive equivalence results considering a consumption tax in representative-agent economies with no capital. Using alternative taxes, we prove similar results considering heterogeneous portfolio holdings with both real and nominal assets.

Other papers consider heterogeneous-agent models with sticky wages instead of sticky prices (Hagedorn et al., 2019, Auclert et al., 2024 or Alves and Violante, 2023). This avoids the countercyclical profits generated by the sticky-price assumption. Sticky wages are formalized using the labor market model Erceg et al. (2000) developed in a representative-agent economy. The normative implication of sticky wages in representative-agent economies has been analyzed in Chugh (2006) and Galí (2015) among others. In particular, it features zero wage inflation in the case of technology shocks. To our knowledge, our paper is the first to derive optimal monetary policy in a HANK model with sticky wages.

This paper also connects to the literature investigating optimal policies with incomplete markets and heterogeneous agents. A first strand of this literature relies on tractable models featuring a simple distribution of wealth, which enables identifying the trade-offs faced by optimal policies. Challe (2020) solves for optimal monetary policy in a "zero-liquidity" environment with endogenous risk. Bilbiie and Ragot (2021) study optimal monetary policy in a tractable model with limited heterogeneity and money. Bilbiie (2024) analyzes a no-trade equilibrium with two types of agents. McKay and Reis (2021) solve for optimal simple fiscal rules (automatic stabilizers) in a tractable model considering exogenous monetary policy.

A second strand of the literature analyses optimal policies with more general distributions of wealth. This is especially true for the literature on optimal fiscal policy in real economies with incomplete-market and heterogeneous-agent models (Aiyagari et al., 2002; Werning, 2007; Bassetto, 2014; Açikgöz et al., 2022; Dyrda and Pedroni, 2022; LeGrand and Ragot, 2024 among others). In this strand, a couple of recent papers study optimal monetary policy with incomplete insurance-markets. Nuño and Thomas (2022) solve for optimal monetary policy under commitment in an economy with uninsurable idiosyncratic risk, nominal long-term bonds, and costly inflation. They propose a methodology based on the calculus of variation. They show that the optimal policy features inflation front-loading that can be sizable in a time-0 problem, but that inflation volatility is reduced in a timeless perspective. Smirnov (2022) and Dávila and Schaab (2023) build on this framework and solve for the optimal monetary policy in a closed economy. They also find that the optimal deviation from price stability is close to 0. Acharya et al. (2023) solve for optimal monetary policy using the tractability of the CARA-normal environment without capital. They show that countercyclical idiosyncratic risk creates a motive for monetary policy to be redistributive. They focus on a time-0 problem, and in their quantitative applications, the optimal deviation from price stability remains small in magnitude except when the price-adjustment cost becomes very small. Bhandari et al. (2021) quantitatively solve for optimal monetary and fiscal policies in a new-Keynesian model with aggregate shocks. They report a significant deviation from price stability at a long horizon, which is partly immune to the time-0 bias, when fiscal instruments are missing and the initial distribution differs from the steady-state one. This result appears to be sensitive to the chosen calibrations of the slope of the Phillips curve and the distribution of firms' profits. We find however that simple fiscal rules reduce optimal inflation volatility, following a similar calibration strategy. McKay and Wolf (2023) solve for optimal monetary policy by considering a linear-quadratic policy problem. They show that household heterogeneity adds a term to the usual loss function due to distributional motives. Finally, Yang (2023) solves for optimal monetary policy by optimizing the coefficients of a Taylor rule in a model with three redistributive channels for inflation: an expenditure channel (households have different consumption baskets and inflation is heterogeneous across products) and the standard Fisher and earning channels. In this literature, our contribution is to characterize the dynamics of optimal monetary policy in a timeless perspective, and in a setup with capital and occasionally binding credit constraints, with different assumptions concerning fiscal policy and nominal frictions.

The paper is organized as follows. Section 2 presents the sticky-price model. Section 3 presents the optimal policies and our equivalence result in this setup. Section 4 presents the tools to simulate the general model. Section 5 reports our quantitative results in the sticky-price economy. Section 6 presents the findings for the sticky-wage economy. Section 7 concludes.

2 The sticky-price environment

Time is discrete, indexed by $t \geqslant 0$. The economy is populated by a continuum of agents of size 1, distributed on a segment J following a non-atomic measure ℓ: $\ell(J) = 1$. Following Green (1994), we assume that the law of large numbers holds.

2.1 Risk

The sole aggregate shock in the economy affects the technology level. We denote this risk by z, which can take any real value. The economy-wide productivity, denoted Z, is assumed to relate to z through: $Z_t = \exp(z_t)$ at all dates t. The history of aggregate risk up to period t is denoted $z^t := \{z_0, z_1, \ldots, z_t\}$.

In addition to the aggregate shock, agents face an uninsurable idiosyncratic labor productivity shock y assumed to belong to a finite set $\mathcal{Y}$ of strictly positive values. An agent i at any date t can adjust their labor supply, denoted by $l_{i,t}$, and earns the before-tax wage rate $\tilde{w}_t$ per efficient unit. Therefore, the agent's total before-tax wage amounts to $y_{i,t}\tilde{w}_t l_{i,t}$. We assume that the productivity process is a first-order Markov chain with a constant transition matrix, denoted by $(\pi_{yy'})_{y,y'}$, where $\pi_{yy'}$ is the probability to switch from productivity $y \in \mathcal{Y}$ in the current period to productivity $y' \in \mathcal{Y}$ in the following period. The share of agents with productivity y, denoted by S_y, is constant across time and equal to: $S_y = \sum_{\tilde{y}} \pi_{\tilde{y}y} S_{\tilde{y}}$ for all $y \in \mathcal{Y}$. Finally, a history of productivity shocks up to date t is denoted by y^t. Using transition probabilities, we can compute the measure θ_t, such that $\theta_t(y^t)$ represents the share of agents with history y^t in period t.

2.2 Preferences

In each period, the economy has two goods: a consumption good and labor. Households are expected-utility maximizers, and they rank streams of consumption $(c_t)_{t\geq 0}$ and of labor $(l_t)_{t\geq 0}$ according to a time-separable intertemporal utility function equal to $\sum_{t=0}^{\infty} \beta^t U(c_t, l_t)$, where $\beta \in (0, 1)$ is a constant discount factor and $U(c, l)$ is an instantaneous utility function. As is standard in this class of models, we focus on the case where U is separable in consumption and labor and is expressed as:[1] $U(c, l) = u(c) - v(l)$, where $u : \mathbb{R}_+ \to \mathbb{R}$ and $v : \mathbb{R}_+ \to \mathbb{R}$ are twice

[1] Our equivalence result does not depend on this functional form. See Section 3.2 for a general function U.

continuously differentiable and increasing. Furthermore, u is concave, with $u'(0) = \infty$, and v is convex.

2.3 Production

The consumption good Y_t is produced by a unique profit-maximizing representative firm that combines intermediate goods $(y^f_{j,t})_j$ from different sectors indexed by $j \in [0,1]$ using a standard Dixit-Stiglitz aggregator with an elasticity of substitution, denoted $\varepsilon > 0$:

$$Y_t = \left[\int_0^1 y^{f\ \frac{\varepsilon-1}{\varepsilon}}_{j,t} dj \right]^{\frac{\varepsilon}{\varepsilon-1}}.$$

For any intermediate good $j \in [0,1]$, the production $y^f_{j,t}$ is realized by a monopolistic firm and sold at price $p_{j,t}$. The profit maximization for the firm producing the final good implies:

$$y^f_{j,t} = \left(\frac{p_{j,t}}{P_t} \right)^{-\varepsilon} Y_t, \text{ where } P_t = \left(\int_0^1 p^{1-\varepsilon}_{j,t} dj \right)^{\frac{1}{1-\varepsilon}}.$$

The quantity P_t is the price index of the consumption good. Intermediary firms are endowed with a Cobb-Douglas production technology and use labor and capital as production factors. The production technology is such that $\tilde{l}_{j,t}$ units of labor and $\tilde{k}_{j,t}$ units of capital are transformed into $Z_t \tilde{k}^{\alpha}_{j,t} \tilde{l}^{1-\alpha}_{j,t}$ units of intermediate good. In equilibrium, this production will exactly cover the demand $y^f_{j,t}$ for the good j, which will be sold with the real price $p_{j,t}/P_t$. We denote by $\tilde{w}_t$ the real before-tax wage rate per efficient unit and $\tilde{r}^K_t$ the real before-tax net interest rate on capital – which are both identical for all firms. The capital depreciation rate is denoted $\delta > 0$. Since intermediate firms have market power that they internalize, the firm's objective is to minimize production costs, including capital depreciation, subject to producing the demand $y^f_{j,t}$. The cost function $C_{j,t}$ of firm j is therefore $C_{j,t} = \min_{\tilde{l}_{j,t}, \tilde{k}_{j,t}} \{(\tilde{r}^K_t + \delta)\tilde{k}_{j,t} + \tilde{w}_t \tilde{l}_{j,t}\}$, subject to $y^f_{j,t} = Z_t \tilde{k}^{\alpha}_{j,t} \tilde{l}^{1-\alpha}_{j,t}$. Denoting by $\zeta_{j,t}$ the Lagrange multiplier of the production constraint, first-order conditions (FOCs) imply:

$$\tilde{r}^K_t + \delta = \zeta_{j,t} \alpha \frac{y^f_{j,t}}{\tilde{k}_{j,t}} \text{ and } \tilde{w}_t = \zeta_{j,t}(1-\alpha) \frac{y^f_{j,t}}{\tilde{l}_{j,t}}. \tag{1}$$

The optimum thus features a common value, denoted by ζ_t, independent of firm type j, with:

$$\zeta_t = \frac{1}{Z_t} \left(\frac{\tilde{r}^K_t + \delta}{\alpha} \right)^{\alpha} \left(\frac{\tilde{w}_t}{1-\alpha} \right)^{1-\alpha}. \tag{2}$$

Firm j's cost is then $C_j = \zeta_t y^f_{j,t}$, which is linear in the demand $y^f_{j,t}$. Following the literature, we assume the presence of a subsidy τ^Y on the total cost, which will compensate for steady-state distortions, such that the total cost supported by firm j is $\zeta_t y^f_{j,t}(1 - \tau^Y)$. Integrating factor price equations (1) over all firms leads to:

$$K_{t-1} = \frac{1}{Z_t}\left(\frac{\tilde{r}_t^K + \delta}{\alpha}\right)^{\alpha-1}\left(\frac{\tilde{w}_t}{1-\alpha}\right)^{1-\alpha} Y_t \text{ and } L_t = \frac{1}{Z_t}\left(\frac{\tilde{r}_t^K + \delta}{\alpha}\right)^{\alpha}\left(\frac{\tilde{w}_t}{1-\alpha}\right)^{-\alpha} Y_t, \tag{3}$$

where Y_t is total production, for which (3) satisfies:

$$Y_t = Z_t K_{t-1}^{\alpha} L_t^{1-\alpha} = \frac{(\tilde{r}_t^K + \delta)K_{t-1} + \tilde{w}_t L_t}{\zeta_t}. \tag{4}$$

Finally, in this setup, the usual factor price relationships do not hold, but we still have:

$$\frac{K_{t-1}}{L_t} = \frac{\alpha}{1-\alpha}\frac{\tilde{w}_t}{\tilde{r}_t^K + \delta}. \tag{5}$$

In a real setup (featuring $\zeta_t = 1$ for all t), equations (2) and (5) fall back to the standard definitions of factor prices: $\tilde{r}_t + \delta = \alpha Z_t(\frac{K_{t-1}}{L_t})^{\alpha-1}$ and $\tilde{w}_t = (1-\alpha)Z_t(\frac{K_{t-1}}{L_t})^{\alpha}$.

In addition to the production cost, intermediate firms face a quadratic price adjustment cost *à la* Rotemberg (1982) when setting their price. Following the literature, the price adjustment cost is proportional to the magnitude of the firm's relative price change and equal to $\frac{\kappa}{2}\left(\frac{p_{j,t}}{p_{j,t-1}} - 1\right)^2 Y_t$, where $\kappa \geq 0$. We can thus deduce the real profit, denoted $\Omega_{j,t}$, at date t of firm j:

$$\Omega_{j,t} = \left(\frac{p_{j,t}}{P_t} - \left(\frac{\tilde{r}_t + \delta}{\alpha}\right)^{\alpha}\left(\frac{\tilde{w}_t}{1-\alpha}\right)^{1-\alpha}\frac{1-\tau^Y}{Z_t}\right)\left(\frac{p_{j,t}}{P_t}\right)^{-\varepsilon} Y_t - \frac{\kappa}{2}\left(\frac{p_{j,t}}{p_{j,t-1}} - 1\right)^2 Y_t - t_t^Y, \tag{6}$$

where t_t^Y is a lump-sum tax financing the subsidy τ^Y. Computing firm j's intertemporal profit requires us to define the firm's pricing kernel. In a heterogeneous agent economy, there is no obvious choice for the pricing kernel. We discuss this in Section 2.7. For the moment, we assume only that the firm's j pricing kernel is independent of its type, and we denote it by $\frac{M_t}{M_0}$. With this notation, firm j's program, which consists in choosing the price schedule $(p_{j,t})_{t\geq 0}$ to maximize the intertemporal profit at date 0, can be expressed as: $\max_{(p_{j,t})_{t\geq 0}} \mathbb{E}_0[\sum_{t=0}^{\infty} \beta^t \frac{M_t}{M_0}\Omega_{j,t}]$. Since this program yields a solution independent of the firm's type j, all firms in the symmetric equilibrium will charge the same price: $p_{j,t} = P_t$. Denoting the gross inflation rate as $\Pi_t^P = \frac{P_t}{P_{t-1}}$ and setting $\tau^Y = \frac{1}{\varepsilon}$ to obtain an efficient steady state, we obtain the standard equation characterizing the Phillips curve in our environment:

$$\Pi_t^P(\Pi_t^P - 1) = \frac{\varepsilon - 1}{\kappa}(\zeta_t - 1) + \beta\mathbb{E}_t\Pi_{t+1}^P(\Pi_{t+1}^P - 1)\frac{Y_{t+1}}{Y_t}\frac{M_{t+1}}{M_t}. \tag{7}$$

The real profit is independent of the firm's type and can be expressed as follows:

$$\Omega_t = \left(1 - \zeta_t - \frac{\kappa}{2}(\Pi_t^P - 1)^2\right) Y_t. \tag{8}$$

2.4 Assets

Agents have the possibility to trade two assets. The first one is nominal public debt, whose supply size is denoted by B_t at date t. Public debt is issued by the government and is assumed to be exempt of default risk. The nominal debt pays off a nominal gross and pre-tax interest rate that is predetermined. In other words, the nominal interest rate between dates $t-1$ and t is known at $t-1$. We denote this (gross and before tax) nominal interest rate by $\tilde{R}^N_{t-1}$. The associated real before-tax (gross) interest rate for public debt is $\tilde{R}^N_{t-1}/\Pi^P_t$. Note that due to inflation, this ex-post real rate is not predetermined. We denote by $b_{i,t}$ the debt investment of agent i. We assume that agents face nominal borrowing constraints, and their nominal debt holdings must be higher than $-\bar{b} \leq 0$. In the rest of the paper, we will focus on the case where the credit limit is above the steady-state natural borrowing limit.[2]

The second asset consists of capital shares, which pay off a (net and before-tax) real interest rate $\tilde{r}^K_t$ – as introduced above. We denote by $k_{i,t}$ the share of capital held by agent i. We assume that agents cannot issue any real claim and real asset holdings must remain positive.

2.5 Government, fiscal tools and monetary policy

In each period t, the government has to finance an exogenous public good expenditure G_t, as well as lump-sum transfers $T_t \geq 0$.[3] These transfers can be thought of as social transfers, which can generate progressivity in the overall tax system. Heathcote et al. (2017) have shown that such transfers are needed to properly replicate the US fiscal system. The government has several tools for financing these expenditures. First, the government can rely on four different taxes. The first two taxes, whose rates are denoted by τ^K_t and τ^B_t, are distortionary taxes, which are levied on real and nominal asset payoffs respectively. Second, the government also fully taxes the firms' profits, which limits the distortions implied by profit distribution. Finally, the last tax, whose rate is denoted by τ^L_t, concerns labor income. In addition to these taxes, the government can also issue a one-period public nominal bond; the amount of public debt outstanding at date t is denoted by B_t. To sum up, fiscal policy is characterized by five instruments $(\tau^L_t, \tau^K_t, \tau^B_t, T_t, B_t)_{t\geq 0}$ given an exogenous path of public spending $(G_t)_{t\geq 0}$.

After-tax quantities are denoted without a tilde. The real after-tax wage rate w_t, as well as the real after-tax interest rates r^K_t, and R^N_t (for capital and public debt, respectively) are as

[2] Aiyagari (1994) discusses the relevant values of the natural borrowing limit in an economy without aggregate shocks. Shin (2006) provides a similar discussion in the presence of aggregate shocks. A standard value in the literature is a zero borrowing limit, which ensures that consumption remains positive in all states of the world.

[3] We rule out the possibility of lump-sum taxes as a standard assumption in this literature (Aiyagari et al., 2002). See Bhandari et al. (2017) for an analysis of the case, where lump-sum taxes can be the planner's instruments.

such expressed as follows:

$$w_t = (1-\tau_t^L)\tilde{w}_t, \tag{9}$$

$$r_t^K = (1-\tau_t^K)\tilde{r}_t^K, \quad \frac{R_t^N}{\Pi_t^P} - 1 = (1-\tau_t^B)(\frac{\tilde{R}_{t-1}^N}{\Pi_t^P} - 1), \tag{10}$$

Taxes on interest-bearing assets are asset-specific and are levied on real returns. The period-t real return on the nominal interest rate $\tilde{R}_{t-1}^N$, set in period $t-1$, is affected by both the period-t inflation rate and the period-t tax τ_t^B (hence the notation R_t^N in (10)).

The government uses its financial resources, comprised of labor and asset tax revenue and public debt issuance, to finance public goods, lump-sum transfers, and debt repayment:

$$G + \frac{\tilde{R}_{t-1}^N}{\Pi_t^P} B_{t-1} + T_t \leq \tau_t^L \tilde{w}_t L_t + \tau_t^K \tilde{r}_t^K K_{t-1} + \tau_t^B \left(\frac{\tilde{R}_{t-1}^N}{\Pi_t^P} - 1 \right) B_{t-1} + \Omega_t + B_t.$$

The expression of this government budget constraint can be simplified, following Chamley (1986). Using the relationships (1) and (4), as well as the definition of post-tax rates in equations (9) and (10), the governmental budget constraint becomes:

$$G + \frac{R_t^N}{\Pi_t^P} B_{t-1} + r_t^K K_{t-1} + w_t L_t + T_t = B_t + \left(1 - \frac{\kappa}{2}(\Pi_t^P - 1)^2\right) Y_t - \delta K_{t-1}. \tag{11}$$

Monetary policy consists in choosing the nominal interest rate $\tilde{R}_t^N$ on public debt (between t and $t+1$), which affects the gross inflation rate Π_t^P. The choice of optimal monetary-fiscal policy is thus a choice of the path of the instruments $(\tau_t^L, \tau_t^K, \tau_t^B, T_t, B_t, \tilde{R}_t^N, \Pi_t^P)_{t\geq 0}$. These instruments are not independent of each other and are intertwined through the budget constraint of the government and the Phillips curve.

2.6 Agents' program, resource constraints, and equilibrium definition

Each agent i is endowed at date 0 by an initial real and nominal wealth $k_{i,-1}$ and $b_{i,-1}$ and an initial productive status $y_{i,0}$. At future dates, an agent's nominal savings pay off the post-tax gross interest rate $\frac{R_t^N}{\Pi_t^P}$ between period $t-1$ and period t, while their real savings pay off the post-tax gross rate $1 + r_t^K$. Formally, the agent's program can be expressed, for given initial endowments $k_{i,-1}$ and $b_{i,-1}$ and given initial shocks $y_{i,0}$ and z_0, as:

$$\max_{\{c_{i,t}, l_{i,t}, k_{i,t}, b_{i,t}\}_{t=0}^{\infty}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(u(c_{i,t}) - v(l_{i,t})\right), \tag{12}$$

$$c_{i,t} + k_{i,t} + b_{i,t} = (1 + r_t^K) k_{i,t-1} + \frac{R_t^N}{\Pi_t^P} b_{i,t-1} + w_t y_{i,t} l_{i,t} + T_t, \tag{13}$$

$$b_{i,t} \geq -\bar{b},\ k_{i,t} \geq 0,\ c_{i,t} > 0,\ l_{i,t} > 0, \tag{14}$$

where $\mathbb{E}_0$ is an expectation operator over both idiosyncratic and aggregate risk.

Equation (13) can be used to discuss the effect of fiscal and monetary policies in relationship to the literature. First, an unexpected change in an asset tax (τ_t^K or τ_t^B) affects period t income, proportionally to interest payments on past real or nominal savings payoffs ($\tilde{r}_t^K k_{i,t-1}$ or $(\frac{\tilde{R}_{t-1}^N}{\Pi_t^P} - 1)b_{i,t-1}$ respectively). Past savings payoffs are the amount of unhedged interest rate exposures (UREs) at period $t-1$, using the wording of Auclert (2019). Second, an unexpected change in inflation affects the return on the nominal holdings, $\frac{R_t^N}{\Pi_t^P}$, due to a Fisher effect. Third, labor tax affects the post-tax wage rate, which generates heterogeneous income and labor-supply effects. Finally, a change in the lump-sum transfer uniformly affects total income without distortion.

At date 0, an agent decides their plans for consumption $(c_{i,t})_{t\geq 0}$, labor supply $(l_{i,t})_{t\geq 0}$, and saving $(b_{i,t})_{t\geq 0}$ and $(k_{i,t})_{t\geq 0}$, which maximize their intertemporal utility (12) subject to the budget constraint (13) and the borrowing limits and positivity constraints (14). These decisions are functions of the initial state $(b_{i,-1}, k_{i,-1}, y_{i,0})$, of the history of the idiosyncratic shock y_i^t and of the history of the aggregate shocks z^t. Thus, there exist sequences of functions, defined over $([-\bar{b}; +\infty) \times [0; +\infty) \times \mathcal{Y}) \times \mathcal{Y}^t \times \mathbb{R}^t$ and denoted by $(c_t, b_t, k_t, l_t)_{t\geq 0}$, such that an agent's optimal decisions can be written as follows:[4]

$$\begin{aligned} c_{i,t} &= c_t((b_{i,-1}, k_{i,-1}, y_{i,0}), y_i^t, z^t),\ b_{i,t} = b_t((b_{i,-1}, k_{i,-1}, y_{i,0}), y_i^t, z^t), \\ k_{i,t} &= k_t((b_{i,-1}, k_{i,-1}, y_{i,0}), y_i^t, z^t),\ l_{i,t} = l_t((b_{i,-1}, k_{i,-1}, y_{i,0}), y_i^t, z^t). \end{aligned} \tag{15}$$

In what follows, we simplify the notation and keep the i-index. For instance, we write $c_{i,t}$ instead of $c_t((b_{i,-1}, k_{i,-1}, y_{i,0}), y_i^t, z^t)$.

The FOCs corresponding to the agent's program (12)–(14) are:

$$u'(c_{i,t}) = \beta \mathbb{E}_t \Big[(1 + r_{t+1}^K) u'(c_{i,t+1}) \Big] + \nu_{i,t}^k, \tag{16}$$

$$= \beta \mathbb{E}_t \left[\frac{R_{t+1}^N}{\Pi_{t+1}} u'(c_{i,t+1}) \right] + \nu_{i,t}^b, \tag{17}$$

$$v'(l_{i,t}) = w_t y_{i,t} u'(c_{i,t}), \tag{18}$$

where the discounted Lagrange multipliers of the real and nominal credit constraints of agent i are denoted by $\nu_{i,t}^k$ and $\nu_{i,t}^b$, respectively. Each of these Lagrange multipliers is null when agent i is not credit-constrained along the instrument under consideration.

[4]The existence of such functions is proven in Miao (2006), Cheridito and Sagredo (2016), and Açikgöz (2018).

We now express the economy-wide constraints:

$$\int_i b_{i,t}\ell(di) = B_t, \quad \int_i k_{i,t}\ell(di) = K_t, \quad \int_i y_{i,t}l_{i,t}\ell(di) = L_t, \tag{19}$$

$$\int_i c_{i,t}\ell(di) + G_t + K_t = \left(1 - \frac{\kappa}{2}(\Pi_t^P - 1)^2\right) Y_t + K_{t-1} - \delta K_{t-1}, \tag{20}$$

which correspond to the clearing of the nominal and real financial markets, of the labor market, and of the goods market, respectively.[5]

Equilibrium definition. Our definition of a market equilibrium can be stated as follows.

Definition 1 (Sequential equilibrium) *A sequential competitive equilibrium is a collection of individual functions* $(c_{i,t}, l_{i,t}, b_{i,t}, k_{i,t}, \nu_{i,t}^b, \nu_{i,t}^k)_{t\geq 0, i\in\mathcal{I}}$, *aggregate quantities* $(K_t, L_t, Y_t, \Omega_t, \zeta_t)_{t\geq 0}$, *price processes* $(w_t, r_t^K, R_t^N, \tilde{w}_t, \tilde{r}_t^K, \tilde{R}_t^N)_{t\geq 0}$, *fiscal policies* $(\tau_t^L, \tau_t^K, \tau_t^B, B_t, T_t)_{t\geq 0}$, *and monetary policies* $(\Pi_t^P)_{t\geq 0}$ *such that, for an initial wealth and productivity distribution* $(b_{i,-1}, k_{i,-1}, y_{i,0})_{i\in\mathcal{I}}$, *and for initial values of capital stock and public debt verifying* $K_{-1} = \int_i k_{i,-1}\ell(di)$ *and* $B_{-1} = \int_i b_{i,-1}\ell(di)$, *and for an initial value of the aggregate shock* z_0, *we have:*

1. *given prices, the functions* $(c_{i,t}, l_{i,t}, b_{i,t}, k_{i,t}, \nu_{i,t}^b, \nu_{i,t}^k)_{t\geq 0, i\in\mathcal{I}}$ *solve the agent's optimization program in equations (12)–(14);*
2. *financial, labor, and goods markets clear at all dates: for any* $t \geq 0$, *equations (19) and (20) hold;*
3. *the government budget is balanced at all dates: equation (11) holds for all* $t \geq 0$;
4. *factor prices* $(w_t, r_t^K, R_t^N, \tilde{w}_t, \tilde{r}_t^K, \tilde{R}_t^N)_{t\geq 0}$ *are consistent with condition (5), as well as with post-tax definitions (9) and (10);*
5. *firms' profits* Ω_t *and the quantity* ζ_t *are consistent with equations (2) and (8);*
6. *the inflation path* $(\Pi_t^P)_{t\geq 0}$ *is consistent with the Phillips curve: at any date* $t \geq 0$, *equation (7) holds.*

2.7 The Ramsey problem

The goal of this paper is to determine the optimal monetary-fiscal policy that generates the sequential equilibrium which maximizes an explicit aggregate welfare criterion. This is a difficult question, as monetary-fiscal policy is composed of seven instruments $(\tau_t^L, \tau_t^K, \tau_t^B, T_t, B_t, \tilde{R}_t^N, \Pi_t^P)_{t\geq 0}$ which affect the saving decisions and the labor supplies of all agents, the capital stock, and price

[5] It would be equivalent to use the sequential representation and to integrate over initial states (of measure Λ) and idiosyncratic histories (of measure θ). For instance, nominal savings can be written as: $\int_i b_{i,t}\ell(di) = \sum_{y_i^t\in\mathcal{Y}^t}\sum_{y_0\in\mathcal{Y}}\int_{b_{-1}\in[-\bar{b},+\infty)}\int_{k_{-1}\in[0,+\infty)} b_t((b_{-1}, k_{-1}, y_0), y_i^t, z^t)\theta_t(y_i^t)\Lambda(db_{-1}, dk_{-1}, y_0) = B_t$.

dynamics. Interestingly, monetary policy has to balance the cost of output destruction (through price adjustment costs), nominal debt monetization, and the indirect effect on mark-ups.

Aggregate welfare. We consider a utilitarian welfare function, where all agents are equally weighted. Formally, the aggregate welfare criterion can be expressed as follows:

$$W_0 := \mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta^t \int_i \left(u(c_{i,t}) - v(l_{i,t}) \right) \ell(di) \right]. \tag{21}$$

Choosing the pricing kernel. In a heterogeneous-agent economy, there is no straightforward choice for the firm's pricing kernel. We follow Bhandari et al. (2021) and Acharya et al. (2023) and assume that firm's pricing kernel is risk-neutral. More precisely, the pricing kernel M_t is constant and normalized to 1 for all t: $M_t := 1$. As noted by others (Bhandari et al., 2021) and as we checked ourselves, the choice of the pricing kernel has minor quantitative effects.[6]

The Ramsey program. The Ramsey program can be expressed using post-tax notation as:

$$\max_{(\tau_t^L, \tau_t^K, \tau_t^B, B_t, T_t, \Pi_t^P, w_t, r_t^K, R_t^N, \tilde{w}_t, \tilde{r}_t^K, \tilde{R}_t^N, K_t, L_t, Y_t, \Omega_t, \zeta_t, (c_{i,t}, l_{i,t}, b_{i,t}, k_{i,t}, \nu_{i,t}^b, \nu_{i,t}^k)_i)_{t \geq 0}} W_0, \tag{22}$$

$$G + \frac{R_t^N}{\Pi_t^P} B_{t-1} + r_t^K K_{t-1} + w_t L_t + T_t = B_t + \left(1 - \frac{\kappa}{2} (\Pi_t^P - 1)^2\right) Y_t - \delta K_{t-1}, \tag{23}$$

$$\text{for all } i \in \mathcal{I}\text{: } c_{i,t} + k_{i,t} + b_{i,t} = (1 + r_t^K) k_{i,t-1} + \frac{R_t^N}{\Pi_t^P} b_{i,t-1} + w_t y_{i,t} l_{i,t} + T_t, \tag{24}$$

$$b_{i,t} \geq -\bar{b}, \nu_{i,t}^b (b_{i,t} + \bar{b}) = 0, \ \nu_{i,t}^b \geq 0, \tag{25}$$

$$k_{i,t} \geq 0, \nu_{i,t}^k k_{i,t} = 0, \ \nu_{i,t}^k \geq 0, \tag{26}$$

$$u'(c_{i,t}) = \beta \mathbb{E}_t \left[(1 + r_{t+1}^K) u'(c_{i,t+1}) \right] + \nu_{i,t}^k, \tag{27}$$

$$u'(c_{i,t}) = \beta \mathbb{E}_t \left[\frac{R_{t+1}^N}{\Pi_{t+1}^P} u'(c_{i,t+1}) \right] + \nu_{i,t}^b, \tag{28}$$

$$v'(l_{i,t}) = w_t y_{i,t} u'(c_{i,t}), \tag{29}$$

$$\Pi_t^P (\Pi_t^P - 1) = \frac{\varepsilon - 1}{\kappa} (\zeta_t - 1) + \beta \mathbb{E}_t \left[\Pi_{t+1}^P (\Pi_{t+1}^P - 1) \frac{Y_{t+1}}{Y_t} \right], \tag{30}$$

$$B_t = \int_i b_{i,t} \ell(di), \quad K_t = \int_i k_{i,t} \ell(di), \quad L_t = \int_i y_{i,t} l_{i,t} \ell(di), \tag{31}$$

and subject to other constraints (which are not reported here for space constraints): the definition (2) of ζ_t, the definition (4) of Y_t, the definition (8) of profits Ω_t, the factor-price relationship (5), the definitions (9)–(10) of post-tax quantities, the positivity of labor and consumption choices, and initial conditions.

[6] We also considered a firm's pricing kernel defined based on the weighted marginal utilities of agents: $M_t := \int_i u'(c_{i,t}) \ell(di)$. This has very small impacts on our results.

The constraints in the Ramsey program include: the governmental and individual budget constraints (23) and (24), the two individual credit constraints on nominal and real asset holdings (and related constraints on $\nu^k_{i,t}$ and $\nu^b_{i,t}$) (25) and (26), Euler equations for consumption and labor (27), (28), (29), the Phillips curve (30), and market clearing conditions for financial and labor markets (31). To simplify the derivation of FOCs, we use some aspects of the methodology of Marcet and Marimon (2019), which is sometimes called the Lagrangian method (Golosov et al., 2016), applied to incomplete-market environments. We denote by $\beta^t\lambda_{i,k,t}$, $\beta^t\lambda_{i,b,t}$, and $\beta^t\lambda_{i,l,t}$ the Lagrange multipliers of the Euler equations (27)–(29) of agent i at date t. Similarly, we denote by $\beta^t\gamma_t$ the Lagrange multiplier of equation (30) of the Phillips curve. The Lagrange multiplier of the government budget constraint is $\beta^t\mu_t$.

The Ramsey program (22)–(31) could be simplified further by following Chamley (1986) and removing taxes and pre-tax quantities, such that the planner directly chooses post-tax quantities. Pre-tax quantities can then be deduced from their definitions (5) and (8). The definitions (9)–(10) of post-tax quantities can be used to recover tax paths from pre- and post-tax quantities. However, we will also consider economies where some fiscal instruments are kept fixed. We have thus chosen to provide the expression of the full-fledged version of the program, where fixing taxes only involves adding a constraint to the program.

As a final remark, the Ramsey program can be written in a recursive form. The state space for the planner and all agents is actually very large. It is the joint distribution over past values of Lagrange multipliers, wealth, productivity levels and the current aggregate state. This inclusion of past values of Lagrange multipliers, which makes the problem difficult, stems from the commitment of the planner not to surprise agents, such that their Euler equations hold in expectation. We omit this representation as the discussion of FOCs in the sequential representation is more intuitive.

3 Optimal policies

This section presents our main equivalence result and analyzes fiscal and monetary policies in different institutional setups. The analysis is greatly simplified if one introduces a new concept, the *social valuation of liquidity (SVL) for agent i,* denoted by $\psi_{i,t}$, and formally defined as:

$$\psi_{i,t} := \underbrace{u'(c_{i,t})}_{\text{direct effect}} - \underbrace{\left(\lambda_{i,k,t} - (1+r^K_t)\lambda_{i,k,t-1}\right) u''(c_{i,t})}_{\text{effect on real savings}} \\ - \underbrace{\left(\lambda_{i,b,t} - \frac{R^N_t}{\Pi^P_t}\lambda_{i,b,t-1}\right) u''(c_{i,t})}_{\text{effect on nominal savings}} + \underbrace{\lambda_{i,l,t} y_{i,t} w_t u''(c_{i,t})}_{\text{effect on labor supply}}. \tag{32}$$

The valuation $\psi_{i,t}$ measures the benefit – from the planner's perspective – of transferring one extra unit of consumption to agent i. This can be interpreted as the planner's version of the marginal utility of consumption for the agent.[7] As can be seen in equation (32), this valuation consists of four terms. The first is the marginal utility of consumption $u'(c_{i,t})$, which is the private valuation of liquidity for agent i. The three other terms can be understood as the internalization, by the planner, of the economy-wide externalities of this extra consumption unit. More precisely, the second and third terms in (32) take into consideration the impact of the extra consumption unit on saving incentives from periods $t-1$ to t and from periods t to $t+1$. The interpretation of both terms is similar, except that they involve real and nominal savings respectively. An extra consumption unit makes the agent more willing to smooth their consumption between periods t and $t+1$, and thus makes their Euler equation (either nominal or real) more "binding". This more "binding" constraint reduces utility by the algebraic quantity $u''(c_{i,t})\lambda_{i,x,t}$, where $\lambda_{i,x,t}$ is the Lagrange multiplier of the agent's Euler equation at date t (whether it's nominal with $x=b$ or real with $x=k$). The extra consumption unit at t also makes the agent less willing to smooth her consumption between periods $t-1$ and t and therefore "relaxes" the constraint at date $t-1$. This is reflected in the quantity $\lambda_{i,x,t-1}$ $(x=b,k)$. In the absence of savings externalities (as in the representative-agent case), the Lagrange multipliers $\lambda_{i,x,t}$ would be 0.

Finally, the fourth term reflects the wealth effect of the labor supply. Indeed, transferring an extra consumption unit to agent i deters their labor supply incentives through their labor supply Euler equation (18). This effect has to be internalized by the planner. Note that this term is present because we have chosen a utility function that is separable in labor and consumption. It would be absent with a GHH utility function.

Another key quantity is μ_t, the Lagrange multiplier on the governmental budget constraint. The quantity μ_t represents the marginal cost in period t of transferring one extra unit of consumption to households. Therefore, the quantity $\psi_{i,t}-\mu_t$ can be interpreted as the "net" SVL: this is from the planner's perspective, the benefit of transferring one extra unit of consumption to agent i, net of the governmental cost. We thus define:

$$\hat{\psi}_{i,t} := \psi_{i,t} - \mu_t. \tag{33}$$

The interpretation of FOCs is greatly clarified by expressing them using $\hat{\psi}_{i,t}$ rather than the multiplier on Euler equations, $\lambda_{i,t}$.

3.1 The flexible-price economy

Our main result below is that the planner reproduces the flexible-price allocations, if they are allowed to choose capital and labor taxes. We thus first analyze the flexible-price allocation, in which the price adjustment cost is $\kappa=0$.[8] In this case, the Phillips curve does not constrain

[7]To simplify the notation, we keep the index i, but the sequential representation can be derived along the lines of equation (15).

[8]Within the literature on optimal fiscal policy in heterogeneous agent models (Werning, 2007; Bassetto, 2014; Açikgöz et al., 2022; or Dyrda and Pedroni, 2022, among others), the concept of net SVL is new, to the best of our

the planner's choices and its associated Lagrange multiplier is $\gamma_t = 0$. We can therefore follow Chamley (1986) and express all of the planner's constraints in post-tax prices. The Ramsey equilibrium can thus be derived using a narrower set of variables, which simplifies the algebra. Taxes are then recovered from the allocation. The before-tax rates $\tilde{w}_t$ and $\tilde{r}_t^K$ can be deduced from equations (2) and (5) with $\zeta_t = 1$. Taxes τ_t^L and τ_t^K are obtained from the relationships between pre-tax and post-tax rates (9) and (10). Finally, profits are null and the nominal rate $\tilde{R}_{t-1}^N$ and the nominal tax τ_t^B are undetermined.[9]

The resolution of the Ramsey program in Appendix A.2 shows that a solution is characterized by: (i) a real and nominal Euler-like equation for each individual valuation $\hat{\psi}_{i,t}$ (as long as i is unconstrained), and (ii) four FOCs related to the planner's four instruments.

The FOCs with respect to nominal individual savings for non-credit-constrained agents can be written as follows:

$$\hat{\psi}_{i,t} = \beta \mathbb{E}_t \left[R_{t+1}^N \hat{\psi}_{i,t+1} \right]. \tag{34}$$

Constrained agents have no nominal Euler equation, as $b_{i,t} = -\bar{b}$ and $\lambda_{i,b,t} = 0$. Equation (34) states that the net SVL should be smoothed out over time with the post-tax nominal interest rate. It can be interpreted as a Euler-like equation for the planner and generalizes the standard individual Euler equation by taking into account the externalities of saving choices. Similarly, the real Euler-like equation, corresponding to real saving choices of unconstrained agents, can be written as follows:

$$\hat{\psi}_{i,t} = \beta \mathbb{E}_t \left[(1 + r_{t+1}^K) \hat{\psi}_{i,t+1} \right] + \beta \mathbb{E}_t \left[(1 + \tilde{r}_{t+1}^K) \mu_{t+1} \right] - \mu_t, \tag{35}$$

while for real constrained agents, we have $k_{i,t} = 0$ and $\lambda_{i,k,t} = 0$. The differences between the real and nominal Euler-like equations are twofold. First, the Euler equation (35) for $\hat{\psi}_{i,t}$ now involves the post-tax real interest rate instead of the nominal one, as in equation (34). Equation (34) also includes a supplementary smoothing term that involves the Lagrange multiplier μ on the governmental budget constraint. This term comes from the discrepancy between the social valuation of real savings and their cost through the governmental budget constraint. This term is absent in the nominal savings equation (34), because the costs of governmental debt and of households' nominal savings are identical and both equal to R_{t+1}^N.

The third FOC concerns the individual labor supply for each agent i, and it can be expressed as follows:

$$v'(l_{i,t}) + \lambda_{i,l,t} v''(l_{i,t}) = w_t y_{i,t} \left(\hat{\psi}_{i,t} + \mu_t \frac{\tilde{w}_t}{w_t} \right). \tag{36}$$

knowledge. It considerably eases the interpretation of the planner's FOCs and numerical simulations.

[9] Indeed, the government pays the before-tax rate $\tilde{R}_{t-1}^N$ and receives the nominal tax τ_t^B, and both quantities have the same base, which is the previous-period public debt. Hence the government only cares about the post-tax nominal rate, like households.

This equation equalizes the marginal social cost of labor (left-hand side) to its marginal benefit (right-hand side). Similarly to the expression (32) of $\psi_{i,t}$, the marginal social cost of labor involves the private cost $v'(l_{i,t})$ as well as the planner's internalization of the general-equilibrium effect of modifying an individual's labor supply, which operates through the individual labor Euler equation (hence the presence of the multiplier $\lambda_{i,l,t}$). The marginal benefits of an extra unit of labor come from the related increase in individual consumption (through $\hat{\psi}_{i,t}$) and from the higher output and higher labor taxes that relax the governmental budget constraint (through μ_t).

The fourth condition, which concerns the post-tax wage rate w_t, is:

$$\int_i \hat{\psi}_{i,t} y_{i,t} l_{i,t} \ell(di) = -\int_i \lambda_{i,l,t} y_{i,t} u'(c_{i,t}) \ell(di). \tag{37}$$

In the absence of any effect on the labor supply, the planner would choose the wage rate so as to set the aggregate net liquidity value – weighted by the individual labor supply in efficient terms – to zero: $\int_i \hat{\psi}_{i,t} y_{i,t} l_{i,t} \ell(di) = 0$, or equivalently, to equalize the social liquidity valuation to its marginal cost: $\int_i \psi_{i,t} y_{i,t} l_{i,t} \ell(di) = \mu_t \int_i a_{i,t-1} \ell(di)$. However, the planner has also to take into account the general-equilibrium distortions implied by wage variations that channel through all individual labor supplies. These distortions are proportional to the Lagrange multipliers on the labor Euler equations, $\lambda_{i,l,t}$.

The fifth FOC concerns the post-tax nominal interest rate R_t^N and is:

$$\int_i \hat{\psi}_{i,t} b_{i,t-1} \ell(di) = -\int_i \lambda_{i,b,t-1} u'(c_{i,t}) \ell(di). \tag{38}$$

Similarly to equation (37), in the absence of any side effects, the planner would like to set to zero the aggregate net value of liquidity among all agents – weighted by agents' nominal asset holdings. However, the planner also has to factor in the side effects of R_t^N on nominal savings incentives, through the Euler equation. This effect is proportional to the shadow cost of the nominal Euler equation. Note that the sign of this shadow cost depends on the planner's perception of the savings quantity in the economy. It is positive when the planner perceives excess nominal savings in the economy, and negative the other way around (see LeGrand and Ragot, 2022a, for a lengthier discussion). In consequence, for instance, when there is an excess quantity of nominal savings in the economy, the total net valuation of liquidity is negative.

The sixth FOC regarding the post-tax real interest rate r_t^K can be expressed as:

$$\int_i \hat{\psi}_{i,t} k_{i,t-1} \ell(di) = -\int_i \lambda_{i,k,t-1} u'(c_{i,t}) \ell(di). \tag{39}$$

This is the exact parallel of equation (38) but for real savings instead of nominal ones.

Finally, the last FOC, regarding the lump-sum transfer T_t, is:

$$\int_i \hat{\psi}_{i,t}\ell(di) \leq 0. \tag{40}$$

This is an equality when $T_t > 0$, and an inequality when $T_t = 0$. Since there are no distortions implied by the lump-sum transfer, it is set such that the redistributive effect is null.

3.2 The equivalence result

The monetary economy features two complementary market imperfections. The first is the monopoly power of intermediary firms, which can yield a price markup ζ_t different from one. The second is the Rotemberg inefficiency, which prevents firms from setting their prices without incurring costs. The two imperfections are complementary. Indeed, in the absence of Rotemberg inefficiency (i.e., $\kappa = 0$), firms' profit maximization yields $\zeta_t = 1$ and the markup inefficiency vanishes, as can be seen from the Phillips curve in equation (7). Conversely, in the absence of imperfect competition, $\zeta_t = 1$, and the Phillips curve implies that the Rotemberg inefficiency plays no role. The planner's objective – in a monetary setup – therefore includes minimizing the impact of these two inefficiencies.

We now show that linear taxes on real and nominal assets, as well as on labor, are sufficient tools to offset these two inefficiencies along the business cycle, even when agents are heterogeneous. To do so, we first solve for the optimal monetary and fiscal policies when the government has access to a full set of fiscal tools. This program can be written as:

$$\max_{(B_t,T_t,\Pi_t^P,w_t,r_t^K,R_t^N,\tilde{w}_t,\tilde{r}_t^K,K_t,L_t,Y_t,\zeta_t,(c_{i,t},l_{i,t},b_{i,t},k_{i,t})_i)_{t\geq 0}} W_0, \tag{41}$$

subject to the same equations as in the Ramsey program (22)–(31). Four observations are in order. First, we have dropped the taxes τ_t^L, τ_t^K, and τ_t^B from the Ramsey program since, as in the flexible-price case, they can be substituted by post-tax quantities w_t, r_t^K, and R^N (and recovered from the resulting allocation). Second, again as in the flexible-price case, the pre-tax nominal rate $\tilde{R}_t^N$ is also dropped since it does not play any role. Third, the before-tax rates $\tilde{w}_t$ and $\tilde{r}_t^K$ play a role only in the markup coefficient ζ_t of equation (2) and in the factor price equation (5). In other words, the before-tax rates can be recovered from ζ_t and from the allocation. Finally, the markup coefficient only appears in the Phillips curve and can thus be recovered from the path of inflation. Because the planner can optimally choose the post-tax nominal rate R_t^N, inflation only appears in the planner's program through the output destruction term (i.e., $-\kappa(\Pi_t^P - 1)^2/2$) in the government budget constraint. Deviating from null inflation, i.e., from $\Pi_t^P = 1$, solely results in a shrinking of the planner's feasible set. The latter therefore chooses to set the gross inflation rate to 1 at all dates, so as to neutralize the Rotemberg inefficiency. This also makes the price markup inefficiency vanish. In the end, the planner faces the same program – and hence chooses the same allocation – as in the real economy. We summarize this first result in the following proposition.

Proposition 1 (An equivalence result) *When labor and both nominal and real asset taxes are available, the government exactly reproduces the flexible-price allocation and net inflation is null in all periods.*

The proof is in Appendix B. Proposition 1 actually holds in a more general framework, where the period utility is general (and not only separable in consumption and labor). The equivalence result would also hold with different market structures. In particular, distributing the profits to a mutual fund as in Bhandari et al. (2021) would preserve the result if a time-varying tax on the fund payoffs is available. Indeed, a noteworthy aspect of the result is that a capital tax per asset choice is needed, hence one distinct instrument for nominal and real asset holdings.

Following the analysis of Kaplan et al. (2018) and Auclert (2019), monetary policy has direct effects, due to price changes, and indirect effects through general-equilibrium feedback. Proposition 1 states that the effects achieved by monetary policy can be achieved by labor and capital taxation. Loosely speaking, on the one hand, outcomes of the direct effects can be replicated by the linear capital tax, which globally affects the return on all savings. On the other hand, general equilibrium effects, affecting the real wage, can be replicated by the linear labor tax, which creates a wedge between the marginal labor cost of the firm, determining their pricing decision, and the labor income of households, determining their labor supply decisions. Finally, the equivalence result holds from time-0 onwards. As a consequence, it is valid both in a time-0 perspective, when instruments are chosen in the initial period before converging to a long-run equilibrium, and in a timeless perspective, when the economy is running for a long period such that the effects of initial conditions have vanished.

Note that the result of Proposition 1 would not hold anymore if the nominal tax is removed as an independent instrument and, for instance, set equal to the real capital tax. In that case inflation would be used to partly substitute for the absence of a specific nominal instrument. Formally, we could not write the program using post-tax quantities as we did, and pre- and post-tax quantities would not be independent of each other.

The result of Proposition 1 is in the same vein as Correia et al. (2008) and Correia et al. (2013), who also show that one can recover price stability if the planer has access to a time-varying consumption tax in a complete market environment. The inclusion of nominal and real asset taxes (instead of a consumption tax) allows us to connect our result to the literature on optimal capital taxation in the heterogeneous-agent literature.[10]

[10]Correia et al. (2008) analyze an economy without capital and with no heterogeneity in asset holdings. In our environment with capital and heterogeneous asset holdings, we need one tax for each asset. In addition, the inclusion of capital tax may be more relevant quantitatively, at least in the US (see Trabandt and Uhlig, 2011).

4 Simulating economies with optimal monetary policy and sub-optimal fiscal policies

This section introduces additional assumptions in order to simulate the model with an exogenous fiscal system. First, following the literature (Gornemann et al., 2016; Bhandari et al., 2021, among many others), we introduce a risk-neutral mutual fund in Section 4.1 to remove agents' portfolio choice. We specify the fiscal rule in Section 4.2 and present the Ramsey program in Section 4.3. Finally, Section 4.4 presents the truncation theory that is used to simulate the model.

4.1 Introducing a mutual fund

The mutual fund collects all interest-bearing asset payoffs, – i.e., the nominal returns of public debt and the real returns of the capital stock – and issues claims for households to invest in. Fund claims are the only asset in which households can now invest, and the fund's returns to households are taxed. All the theoretical results of the previous section, and in particular the equivalence result of Proposition 1, remain valid with this new market structure.

The (before-tax) interest rate paid by this fund to agents is denoted by $\tilde{r}_t$. The three interest rates, for public debt, capital, and the fund, are connected by two different relationships. The first reflects the non-profit condition of the fund. We denote by A_t the total asset amount in the economy, equal to the sum of public debt and capital: $A_t = K_t + B_t$. Since the fund holds all the public debt and the capital and sell shares, its non-profit condition at date t implies:

$$\tilde{r}_t A_{t-1} = \tilde{r}_t^K K_{t-1} + \left(\frac{\tilde{R}_{t-1}^N}{\Pi_t^P} - 1 \right) B_{t-1}. \tag{42}$$

The second relationship is the no-arbitrage condition between public debt holdings and capital shares. This condition states that one unit of consumption invested in each of the two assets should yield the same expected return. Formally, this condition can be written as:

$$\mathbb{E}_t \left[\frac{\tilde{R}_t^N}{\Pi_{t+1}^P} \right] = \mathbb{E}_t \left[1 + \tilde{r}_{t+1}^K \right]. \tag{43}$$

Because of the fund intermediation, households make no actual portfolio choice, and we will denote by a_t their holdings in fund claims. Agents face borrowing constraints, and their fund holdings must be higher than $-\bar{a} \leq 0$.

With the introduction of the mutual fund, there is only a single tax on the interest payments of fund shares. We still denote τ_t^K this unique capital tax, and the post-tax fund interest rate is:

$$r_t = (1 - \tau_t^K) \tilde{r}_t. \tag{44}$$

With this notation, households' budget constraints and credit limits are:

$$a_{i,t} + c_{i,t} = (1 + r_t)a_{i,t-1} + w_t y_{i,t} l_{i,t} + T_t, \tag{45}$$

$$a_{i,t} \geq -\overline{a}. \tag{46}$$

Because of the absence of portfolio choice, households have a unique consumption Euler equation:

$$u'(c_{i,t}) = \beta \mathbb{E}_t \Big[(1 + r_{t+1})u'(c_{i,t+1})\Big] + \nu_{i,t}, \tag{47}$$

where $\beta^t \nu_{i,t}$ is the Lagrange multiplier on the credit constraint (46). The labor FOC (29) is unchanged. The governmental budget constraint and the financial market clearing conditions are:

$$G + B_{t-1} + r_t A_{t-1} + w_t L_t + T_t = B_t + \left(1 - \frac{\kappa}{2}(\Pi_t^P - 1)^2\right) Y_t - \delta K_{t-1}, \tag{48}$$

$$\int_i a_{i,t} \ell(di) = A_t = K_t + B_t. \tag{49}$$

4.2 The fiscal rule

We consider a fiscal system that has an affine structure, composed of lump-sum transfers and linear marginal tax rates. Such a system has often been used in the literature because it allows one to simply reproduce the redistributivity of the US fiscal system, as shown for instance by Heathcote et al. (2017) and Dyrda and Pedroni (2022). Importantly, this fiscal system is not set optimally, but through ad-hoc fiscal rules. For tax rates, we extend the analysis of Vegh and Vuletin (2015) to the 1960-2023 period and verify that the tax rates on personal and corporate incomes are acyclical in the United States. The empirically relevant case therefore corresponds to maintaining constant marginal tax rates over the business cycle. Our empirical analysis can be found in Appendix C.1.[11] For the lump-sum tax, we follow the standard rule of Bohn (1998), in which the primary budget depends on the deviation of the public debt from its long-run target.

Formally, we assume that taxes are defined through the following rules:

$$\tau_t^L = \tau_*^L, \tag{50}$$

$$\tau_t^K = \tau_*^K, \tag{51}$$

$$T_t = T_* - \sigma^T (Z_t - Z_*) - \sigma^B (B_t - B_*), \tag{52}$$

where T_*, τ_*^L, τ_*^K, B_*, and Z_* are steady-state values of the corresponding variables. Keeping public debt stationary requires $\sigma^B > 0$, and different values for σ^T will imply different elasticities of public debt to TFP.

[11] We consider time-varying tax rates in Appendix C.2 so as to investigate the impacts of a time-varying fiscal system.

4.3 The Ramsey allocation

The Ramsey planner's program can be written as:

$$\max_{\left(w_t,r_t,\tilde{w}_t,\tilde{r}_t^K,\tilde{R}_t^N,T_t,K_t,L_t,\Pi_t^P,(a_{i,t},c_{i,t},l_{i,t},\nu_{i,t})_i\right)_{t\geq 0}} W_0, \tag{53}$$

$$\text{s.t. } \tau_t^K, \tau_t^L, B_t \text{ following the fiscal rules (50)–(52),} \tag{54}$$

and subject to: the governmental budget constraint (48), the household budget constraint (45), the household credit constraint (46), the Euler equation on consumption (47), the FOC on labor (18) – which is unchanged, the Phillips curve (7), the market clearing condition (49), the labor market clearing condition (19), the fund no-profit condition (42), the no-arbitrage condition (43), and the post-tax rate definitions (9) and (44).

Constraints (54) imply that the dynamics of tax rates are exogenous. This is the main difference with Section 3: taxes cannot be adjusted over the business cycle to implement the optimal post-tax real wage and interest rate. For this reason, there is room for inflation to be used to redistribute wealth across agents and to provide insurance against aggregate shocks. Since the algebra to determine the optimal inflation rate with fixed tax rates is more involved than in Section 3, we focus here solely on the intuitions about the trade-offs for the inflation rate. We formally derive all FOCs in Appendix C.2.

Deriving the FOCs of the planner for the choice of Π_t^P allows one to identify three mechanisms:

1. Changing inflation can affect the real wage due to the Phillips curve, which can be useful as an indirect tool to transfer resources across households.

2. Unexpected inflation specifically reduces the real return on public debt, which reduces the return on the fund and decreases real interest payments by the government.

3. Inflation destroys resources due to the adjustment cost.

These effects can be identified analytically from the program of the planner. We use the following notation: μ_t denotes the Lagrange multiplier on the relevant expression of the government budget constraint, γ_t the Lagrange multiplier of the New-Keynesian Phillips curve (7), Γ_t the Lagrange multiplier on the no-profit condition of the fund (42), and Υ_t the Lagrange multiplier on the no-arbitrage condition (43). With this notation, the FOC on inflation can be written as follows:

$$\overbrace{\mu_t \kappa(\Pi_t^P - 1)}^{\text{Marginal cost of inflation}} = \overbrace{(\gamma_{t-1} - \gamma_t)(2\Pi_t^P - 1)}^{\substack{\text{Manipulation of real wage}\\ \text{with NK Phillips curve}}} - \underbrace{\beta^{-1}\Upsilon_{t-1}\frac{\tilde{R}_{t-1}^N}{Y_t\left(\Pi_t^P\right)^2}}_{\text{Change in the nominal interest rate}} \underbrace{+\Gamma_t(1-\tau_t^K)B_{t-1}\frac{\tilde{R}_{t-1}^N}{Y_t\left(\Pi_t^P\right)^2}}_{\text{Change in the fund interest rate}}. \tag{55}$$

Equation (55) sets the marginal cost of inflation (on the left hand side) equal to its marginal benefits (right hand side). The marginal cost of higher inflation is simply related to the destruction of output, proportional to the parameter κ. This marginal cost is zero in the absence of inflation ($\Pi_t^P = 1$). The marginal benefits include three effects that stem from the modification of prices. First, a change in the inflation rate affects the real wage rate through the NK Phillips curve ("manipulation of real wage"). Second, it also affects the level of the nominal interest through the no-arbitrage condition of the fund. Third, inflation also modifies the interest rate of the fund (which is the financial return for households) through the no-profit condition of the fund. This effect is proportional to the size of public debt, which drives how much the nominal interest matters for the return of the fund. The redistributive effects of these price changes are captured by the Lagrange multipliers.

Decreasing the fund interest rate relaxes the fund's no-profit constraint proportionally to its Lagrange multiplier Γ_t. The expression of Γ_t is $\Gamma_t = \mu_t - \frac{1}{A_{t-1}}\int_i \left(\psi_{i,t}a_{i,t-1} + \lambda_{i,t-1}u'(c_{i,t})\right)\ell(di)$ (see equation (86) in Appendix C.2). So, decreasing the fund interest rate relaxes the governmental budget constraint (valued with the multiplier μ_t), and generates redistribution across agents in proportion to their beginning-of-period wealth (term $\int \psi_{i,t}a_{i,t-1}di$); it also affects their saving incentives via the Lagrange multiplier λ_i of the Euler equation (term $\int \lambda_{i,t-1}u'(c_{i,t})di$). These two last terms are negative because we consider a decrease in the interest rate. The other multipliers γ_t and Υ_{t-1} have similar interpretations, which are slightly more involved because of general equilibrium effects (see the FOCs of the Ramsey program in Appendix C.2)

4.4 Simulating the model: The truncation representation

The Ramsey problem of Section 4.3 cannot be solved with simple simulation techniques. Indeed, the Ramsey equilibrium is now a joint distribution across wealth and Lagrange multipliers, which is a high-dimensional object. The steady-state value of the set of Lagrange multipliers is not easy to find, and the planner's instruments depend on the dynamics of this joint distribution. For this reason, we use the truncation method of LeGrand and Ragot (2022a) to determine the joint distribution of wealth and Lagrange multipliers.[12] The accuracy of optimal policies found with this method has been further analyzed in LeGrand and Ragot (2023), for both the steady state and the dynamics.

4.4.1 The uniform truncation method

The intuition of the method can be summarized as follows. In heterogeneous-agent models, agents differ according to their idiosyncratic history. An agent i has a period-t history $y_i^t = \{y_{i,0}, \ldots, y_{i,t}\}$. Let $h = (\tilde{y}_{-N+1}, \ldots, \tilde{y}_{-1}, \tilde{y}_0)$ be a given history of length N. In period t, an agent i is said to have *truncated history* h if the history of this agent for the last N periods is equal to $h = \{y_{i,t-n+1}, \ldots, y_{i,t}\}$. The idea of the truncation method is to aggregate agents having the same truncated history and to express the model using these groups of agents rather than individuals. This generated the so-called *truncated model*, which features a finite state space. The "true" Bewley model features wealth heterogeneity among the agents having the same truncated history h. This simply comes from the heterogeneity in histories prior to the aggregation period (i.e., more than N periods ago). We capture this within-truncated-history heterogeneity through additional parameters – denoted by "ξs" – which are truncated-history specific. This construction yields a finite state-space representation, which is exogenous to agents' choices and thereby allows one to compute optimal policies.[13] The details of the truncated model can be found in Appendix D.

To find the steady-state values of the Lagrange multipliers, we use the same algorithm as in LeGrand and Ragot (2022a):

1. Set a truncation length N and guess values for the planner's instruments.
2. Solve the steady-state allocation of the full-fledged Bewley model with the previous instrument values, using standard techniques.
3. Consider the truncated representation of the economy for a truncation length N.
   (a) Solve for the joint distribution of wealth and Lagrange multipliers.
   (b) Use the Ramsey FOCs to check whether the values of instruments are above or below their optimal value.
4. Change the instruments' values accordingly (or stop if their value is close enough to the optimal value), and redo the process from Step 2.
5. Increase the truncation length N, and restart from Step 2 until increasing N has no impact on the instruments' values.

[12] Optimizing on simple rules in the spirit of Krusell and Smith (1998) is also hard to implement, as the shape of inflation is highly non-linear.

[13] Considering wealth bins is not possible, as the savings function and thus the transitions across wealth bins is endogenous to the planner's policy. This would imply a fixed point which would be very hard to solve.

We analyze optimal monetary policy with a given fiscal policy. This greatly simplifies the process, as it is known that the optimal gross inflation rate is $\Pi^P = 1$. As a consequence, using the previous procedure it is very fast to compute the joint distribution of wealth and Lagrange multipliers.

To use our truncation method in the presence of aggregate shocks, we further assume that:

1. The parameters $(\xi_h)_h$ remain constant and equal to their steady-state values.

2. The set of credit-constrained histories is time-invariant.

We thus assume that the time-variation of within-history heterogeneity is small enough for it to have only a second-order effect. In addition, we assume that the aggregate shock is small enough for the set of credit-constrained histories not to change in the dynamics. Both assumptions rely on the choice of the truncation length and can be checked numerically.

4.4.2 The refined truncation method

The previous truncation method is simple to implement, but it has the drawback that it considers many histories, some of which are very unlikely to be experienced by agents. By the law of large numbers, these histories concern a very small number of agents. For instance, for a truncation length of $N = 5$ used below, our calibration implies that many histories have a size smaller than 10^{-6}. The idea in LeGrand and Ragot (2022b) is to consider different truncation lengths for different histories. For the sake of clarity, we will call this method the *refined* truncation, while the former one will be called the *uniform* truncation. Histories more likely to be experienced (i.e., with a bigger size) can be "refined", which means that they can be substituted by a set of histories with higher truncation lengths. For instance, the truncated history (y_1, y_1) $(N = 2)$ can be refined into $\{(y, y_1, y_1) : y \in \mathcal{Y}\}$, where the group of agents who have been in productivity y_1 for two consecutive period is split into $Card\mathcal{Y}$ truncated histories.

A benefit of this construction is that the number of histories is a *linear* function of the maximum truncation length, instead of an exponential function. The construction of the refined method is detailed in LeGrand and Ragot (2022b). A difficulty of this construction is that the set of refined histories must form a well-defined partition of the set of idiosyncratic histories in each period. To keep the exposition simple, we solve the model with a uniform truncation method, and only use the refinement in Section 5.5 as a robustness check.

5 Quantitative assessment of the sticky-price model

5.1 The calibration and steady-state distribution

Preferences. The period is a quarter. The discount factor is $\beta = 0.99$, and the period utility function is: $\log(c) - \chi^{-1}\frac{l^{1+1/\varphi}}{1+1/\varphi}$.[14] The Frisch elasticity of labor supply is set to $\varphi = 0.5$, which is the value recommended by Chetty et al. (2011) for the intensive margin in heterogeneous-agent models. The scaling parameter is $\chi = 0.076$, to obtain an aggregate labor supply of roughly 1/3.

Technology and TFP shock. The production function is Cobb-Douglas: $Y = ZK^{\alpha}L^{1-\alpha}$. The capital share is set to $\alpha = 36\%$ and the depreciation rate to $\delta = 2.5\%$, as in Krueger et al. (2018) among others. The TFP process is a standard AR(1) process, with $Z_t = \exp(z_t)$ and $z_t = \rho_z z_{t-1} + \varepsilon_t^z$, where $\varepsilon_t^z \overset{\text{iid}}{\sim} \mathcal{N}(0, \sigma_z^2)$. We use the values $\rho_z = 0.95$ and $\sigma_z = 0.31\%$ to obtain a standard deviation of the TFP shock z_t equal to 1% at a quarterly frequency (see Den Haan, 2010, for instance).

Idiosyncratic risk. We use a productivity process: $\log y_t = \rho_y \log y_{t-1} + \varepsilon_t^y$, with $\varepsilon_t^y \overset{\text{iid}}{\sim} \mathcal{N}(0, \sigma_y^2)$. We calibrate a persistence of the productivity process $\rho_y = 0.99$ and a standard deviation of $\sigma_y = 12.3\%$. These values are consistent with empirical evidence (Krueger et al., 2018). This process generates a realistic empirical pattern for wealth. The Gini coefficient of the wealth distribution amounts to 0.73, while the model implies an average annual capital-to-GDP ratio of 2.5. These two values are in line with their empirical counterparts. Finally, the Rouwenhorst (1995) procedure is used to discretize the productivity process into 5 idiosyncratic states with a constant transition matrix.

Fiscal system. The steady-state parameters of the fiscal rules are calibrated based on the computations of Trabandt and Uhlig (2011), who use the methodology of Mendoza et al. (1994) on public finance data prior to 2008. This approach consists in computing a linear tax on capital and on labor, as well as lump-sum transfers that are consistent with the governmental budget constraint. Their estimations for the US in 2007 yield a capital tax (including both personal and corporate taxes) of 36%, a labor tax of 28% and lump-sum transfers equal to 8% of the GDP. We then consider the steady-state values $(\tau_*^L, \tau_*^K, T_*/GDP) = (28\%, 36\%, 8\%)$. This steady-state fiscal system generates two untargeted outcomes. First, it implies a public debt-to-GDP ratio equal to 63.5%, which is very close to the value of 63% estimated by Trabandt and Uhlig (2011). Second, it also implies a public spending-to-GDP ratio equal to 12.1%. This value is consistent with other quantitative investigations (Bhandari et al., 2017), even though a

[14] Note that matching the data constrains the Intertemporal Elasticity of Substitution (IES) parameter. Considering a log utility, we can match a Gini coefficient of wealth of 0.73 close to its empirical counterpart of 0.77. Considering a CRRA function $u(c) = \frac{c^{1-\sigma}}{1-\sigma}$ with $\sigma = 2$ reduces the Gini coefficient to 0.57. A higher value of σ decreases the Gini coefficient, as agents save more to self-insure.

little bit low compared to the postwar value, which has decreased to 14.1% in 2017, from 17% in the 1970s.

Considering the dynamic part of the fiscal system, we implement the fiscal rules of equations (50)–(52). Tax rates are constant and equal to their steady-state values. The two parameters, σ^T and σ^B, of the rule à la Bohn characterize how the lump-sum tax reacts to aggregate shocks and public debt. To set them, we use two constraints. First, we impose that a negative TFP shock of 1% implies an average increase of debt over GDP of 2%, which lies in the range estimated by Kim and Zhang (2021) for developed countries. Second, we also set a rule that ensures a stationary path for public debt. We obtain that our fiscal rule is characterized by the parameters $(\sigma^T, \sigma^B) = (8.5, 4.0)$.

Monetary parameters. We follow the literature and assume that the elasticity of substitution across goods is $\varepsilon = 6$ and the price adjustment cost is $\kappa = 100$ (see Bilbiie and Ragot, 2021, for a discussion and references). This implies a slope of the quarterly Phillips curve of 5%, or an average price duration of 5 quarters in the corresponding Calvo model. Indeed, the slope is $(\varepsilon - 1)/\kappa$ in the Rotemberg model and it is $(1-\theta)(1-\beta\theta)/\theta$ in the Calvo model, where $1/(1-\theta)$ is the average price duration (Galí, 2015). We further discuss the calibration of slope values in Section 5.4. Table 1 provides a summary of the model parameters.

Steady-state equilibrium distribution. We first simulate the full-fledged Bewley model (i.e., without aggregate shocks) with the steady-state optimal inflation rate $\Pi^P = 1$. In Table 2, we report the wealth distribution generated by the model and compare it to the empirical distribution. We compute a number of standard statistics – listed in the first column – including the quintiles, the Gini coefficient, and Top 5% property.

The empirical wealth distribution reported in the second and third columns of Table 2 is computed using two sources, the PSID for the year 2006 and the SCF for the year 2007. The fourth column reports the wealth distribution generated by our model. The Gini coefficient of the model is 0.73, whereas it is 0.77 in the data. The model reproduces well the bottom of the distribution, but doesn't do as well for the very top. It is known that additional model features must be introduced to match the high wealth inequality in the US, such as heterogeneous discount rates, as in Krusell and Smith (1998), entrepreneurship, as in Quadrini (1999), or stochastic returns (Guvenen et al., 2023).

Truncation. We now construct the truncated model. We use a truncation length of $N = 5$, which with our 5 states, implies $5^5 = 3125$ different truncated histories. We only focus on histories with positive size (some histories are never experienced if there is some zero probability to switch from one state to another), which reduces the number of histories to 727. LeGrand and Ragot (2022a), showed that in environments without nominal frictions, a small truncation

Parameter	Description	Value
	Preference and technology	
β	Discount factor	0.99
σ	Curvature utility	1
α	Capital share	0.36
δ	Depreciation rate	0.025
$\bar{a}$	Credit limit	0
χ	Scaling param. labor supply	0.076
φ	Frisch elasticity labor supply	0.5
	Shock process	
ρ_z	Autocorrelation TFP	0.95
σ_z	Standard deviation TFP shock	0.31%
ρ_y	Autocorrelation idio. income	0.99
σ_y	Standard dev. idio. income	12.3%
	Tax system	
τ_*^K	Capital tax	36%
τ_*^L	Labor tax	28%
T_*	Transfer over GDP	8%
	Monetary parameters	
κ	Price adjustment cost	100
ε	Elasticity of sub.	6

Table 1: Parameter values in the baseline calibration. See text for descriptions and targets.

length can yield accurate results, thanks to the introduction of the ξ parameters accounting for within-bin heterogeneity, as explained in Section 4.4.1. In Section 5.5, we show that this is also true in the current setup and use the refined truncation as a robustness check.

5.2 Optimal inflation dynamics

We simulate the model after a transitory negative TFP shock of one standard deviation. In period 0, we assume that the economy and the various Lagrange multipliers are initialized to their steady state values. The simulations thus involve a timeless perspective, and the economy will eventually go back to its steady state. We compare the outcomes of three economies. The fiscal system is common to the three economies and features constant marginal tax rates as detailed above. The three economies differ in their monetary policies. In the benchmark economy (labeled Economy 1), monetary policy is optimally set. In the second economy (Economy 2), the inflation path remains constant at $\Pi_t^P = 1$ for all t. Indeed, as noted in Bhandari et al. (2021), this constant path corresponds to the optimal inflation path for a complete market economy with a TFP shock,

	Data		Model
Wealth statistics	PSID, 06	SCF, 07	
Q1	−0.9	−0.2	0.0
Q2	0.8	1.2	0.2
Q3	4.4	4.6	5.2
Q4	13.0	11.9	18.2
Q5	82.7	82.5	76.4
Top 5%	36.5	36.4	33.2
Gini	0.77	0.78	0.73

Table 2: Wealth distribution in the data and in the model.

due to the "divine coincidence" identified by Woodford (2003). Finally, in the third economy (Economy 3), monetary policy is implemented via a Taylor Rule.[15] Comparing Economies 1 and 2 helps us identify the contribution of market incompleteness to the optimal inflation dynamics, while comparing Economies 1 and 3 helps us to identify the contribution of the optimal policy in reducing inflation volatility. We report in Figure 1 the Impulse Response Functions (IRFs thereafter) for key variables of the three economies, which all start from their steady state value in period 0 (i.e., timeless perspective). In panel 10, wage inflation is defined as the growth rate of the nominal wage, computed using real wage and the price level: $\Pi_t^W = \frac{w_t P_t}{w_{t-1} P_{t-1}} = \Pi_t^P \frac{w_t}{w_{t-1}}$.

First, price inflation (panel 9, reported with a quarterly basis) in Economy 1 moves slightly on impact and then decreases to converge back to its steady-state value. However, the variations are quantitatively modest and inflation volatility is low. The absolute maximum change in the quarterly inflation rate amounts to 0.02%. Since the inflation movements are small in magnitude, the differences between the allocations of Economies 1 and 2 (i.e., optimal inflation rate vs. $\Pi_t^P = 1$) are also small. Regarding the fiscal dimension, tax rates are constant by construction and the elasticity of public debt over GDP to TFP can be verified to be around −2: a fall of TFP by one standard deviation increases public debt ratio by 0.6%. The comparison of Economies 1 and 2 allows us to check that the optimal inflation path provides some insurance against aggregate risk. This will be more clearly detailed in Section 5.3 through welfare comparisons. In Economy 3, the Taylor rule leads to a slight overreaction of inflation that decreases more than with optimal policy. This implies that the drop in GDP is lower and more persistent than in Economy 1. However, the effects are small due to the limited magnitude of the overreaction. Overall, these results are consistent with those of Bhandari et al. (2021), but with a lower inflation volatility

[15] The functional form of the Taylor rule connecting the pre-tax nominal gross rate $\tilde{R}_t^N$ to inflation is standard: $\frac{\tilde{R}_t^N}{\tilde{R}^N} = \left(\frac{\Pi_t^P}{\Pi^P}\right)^{\phi_\Pi}$, where $\tilde{R}^N$ and Π^P are steady-state values and ϕ_Π is the response of the nominal interest rate to inflation. We set $\phi_\Pi = 1.5$, which is also a standard value in the literature (see Galí, 2015).

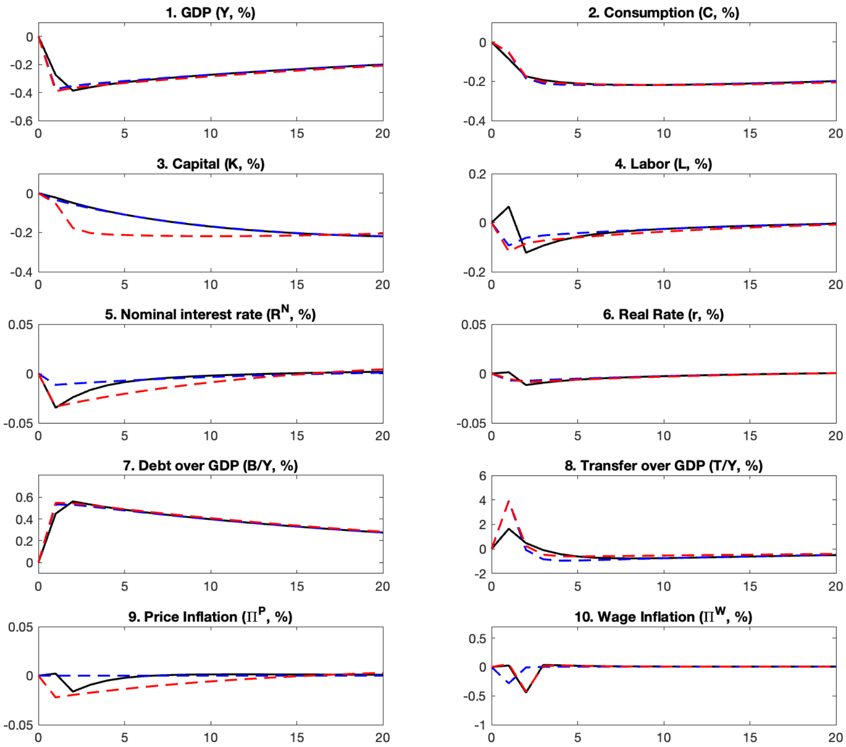

Figure 1: IRFs in percentage points after a negative productivity shock of one standard deviation for relevant variables. The black line is the benchmark economy with optimal monetary policy. The blue dashed line is Economy 2, where we impose $\Pi_t^P = 1$. The red dashed line is Economy 3, where monetary policy follows a Taylor rule.

due to the different calibration strategy.[16] We provide second-order moments of these three economies in Appendix D.6. They confirm that the differences between the three economies are small and that the volatility of consumption is slightly smaller under the optimal monetary policy.

5.3 Welfare comparison

We further investigate the differences between Economies 1 and 2 by computing the welfare gap in terms of consumption equivalents between the two economies. The welfare gap is defined as the constant percentage increase in consumption of Economy 1 (with optimal policy), for the intertemporal welfare of Economy 1 with increased consumption to match the one of Economy 2.

[16]We verify in Section E below that opting for a calibration strategy similar to the one of Bhandari et al. (2021) yields an inflation volatility that is similar to theirs.

Formally, using the utilitarian social welfare function (21), the welfare gap Δ is defined as:

$$\mathbb{E}_0\left[\sum_{t=0}^{\infty}\beta^t\int_i\left(u((1+\Delta)c_{i,t}^{(1)})-v(l_{i,t}^{(1)})\right)\ell(di)\right]=\mathbb{E}_0\left[\sum_{t=0}^{\infty}\beta^t\int_i\left(u(c_{i,t}^{(2)})-v(l_{i,t}^{(2)})\right)\ell(di)\right], \quad (56)$$

where the superscripts (1) and (2) refer to the allocation of Economies 1 and 2, respectively. Since the optimal policy is welfare-improving by definition, we expect the welfare gap to be positive: $\Delta > 0$.

In our environment, and following the discussions of Dyrda and Pedroni (2022) and Bhandari et al. (2023), this computation of welfare gaps captures insurance effects against aggregate risk. Indeed, we compute welfare gaps simulating the effect of two policies in the same economy, starting from the same steady state and hit by the same shocks. The fiscal policies do not change the steady state, and do not implement permanent transfers across agents. As a consequence, the welfare gaps come from the ability of the different policies to provide insurance against aggregate risks.[17]

To better analyze the insurance property, we can condition the welfare gaps on the productivity of each agent. We thus compute the welfare gap Δ_y ($y \in \mathcal{Y}$) conditional on an initial productivity level y as follows:

$$\mathbb{E}_0\left[\sum_{t=0}^{\infty}\beta^t\int_i(u((1+\Delta_y)c_{i,t}^{(1)})-v(l_{i,t}^{(1)}))1_{y_{i,0}=y}\ell(di)\right]=\mathbb{E}_0\left[\sum_{t=0}^{\infty}\beta^t\int_i(u(c_{i,t}^{(2)})-v(l_{i,t}^{(2)}))1_{y_{i,0}=y}\ell(di)\right], \quad (57)$$

where $1_{y_{i,0}=y} = 1$ if the initial state $y_{i,0}$ is y and 0 otherwise. Consistently with the timeless perspective, we implement this welfare comparison assuming that the economy starts from the steady-state distribution. We then simulate the economy for 10,000 periods with 10 different realizations of the aggregate shock path. The intertemporal welfare gaps are computed as an average over these realizations, where we track agents with a given initial productivity level (which then evolves according to the stochastic process for idiosyncratic risk). As discussed above, the difference in the welfare gaps according to the initial productivity level should be understood as a decomposition of the aggregate gaps of insurance across initial types.

We find that implementing the optimal monetary policy – measured as the welfare gap between Economies 1 and 2 – generates an average welfare gain of 0.002%. This small value can be expected from Figure 1 and from the second-order moments of Table 7. However, this value hides heterogeneity among productivity levels, as can be seen in Figure 2, which plots the welfare gaps between Economies 1 and 2 for different productivity levels. If all productivity levels experience a positive welfare gain due the optimal monetary policy, the lower the productivity, the more the agents benefit from the optimal policy. Low productive agents they enjoy a higher

[17] Following Floden (2001) and Benabou (2002), the literature has often decomposed the total welfare gap into three effects: a level effect, an insurance effect, and distributive effects. As we do not consider transitions starting from an initial distribution (as Bhandari et al., 2023, or Dyrda and Pedroni, 2022), we only consider the insurance effects.

reduction in consumption volatility, while more productive agents are able to self-insure, which reduces the welfare gains of optimal policy.

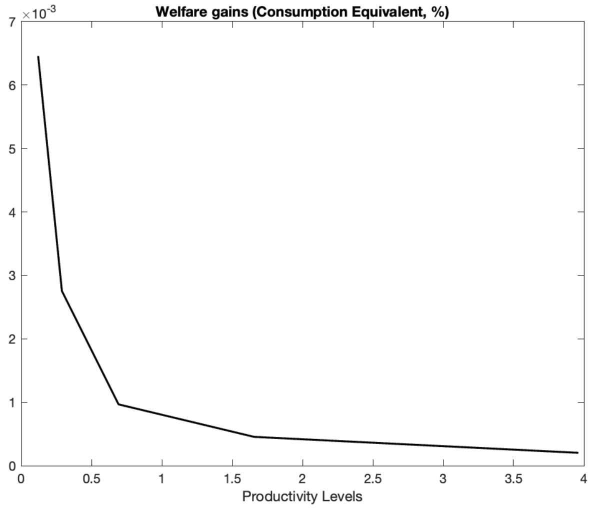

Figure 2: Welfare gains by productivity level associated to optimal monetary policy. The welfare gains are computed using equation (57).

5.4 Understanding the mechanisms

Slope of the Phillips curve. The recent empirical literature offers a wide range of estimates for the price adjustment cost and the implied slope of the Phillips curve. We present these alternative values in Appendix E.1, where we consider slopes varying from the low level of 0.62% of Hazell et al. (2022) to the high one of 23% of Barnichon and Geert (2020). These two values correspond to average price durations in the Calvo model of 14.0 and 2.7 quarters, respectively. The deviation from price stability is small in any case, and nominal wage moves to adjust the real wages.

Role of the distribution of profits. As our analysis adds new ingredients to the existing literature (namely binding credit constraints, capital accumulation, a mutual fund, timeless perspective, and the resolution via the truncation method), we now investigate how our results compare to those of Acharya et al. (2023), Bhandari et al. (2021), or Nuño and Thomas (2022), among others. A first difference is that we consider a timeless perspective and not a time-0 program. As explained in Nuño and Thomas (2022), inflation deviation after a TFP shock is larger in a period-0 problem compared to a timeless perspective, as the planner can surprise agents with an increase in inflation in period 0, to redistribute wealth across agents.

However, some papers, such as Bhandari et al. (2021), find that inflation can sizably increase after a TFP shock in an environment with exogenous fiscal policy, which is immune to the time-0 inconsistency. To understand these results and the role of time-varying fiscal policy, we analyze various models in Appendix E. First, we consider (Section E.2) a simple model, with only two types of agents (called TANK model in the literature). It allows us to identify the key determinants driving the inflation response, which are an unequal profit distribution, a high slope of the Phillips curve, the absence of exogenous time-varying fiscal policy, and to a lesser extent, a low IES of agents. An additional gain of the simple model is that it can be easily related to the quantitative model, with a small truncation length ($N = 1$).

Second, we verify (Section E.3) that the factors identified in the simple model remain valid in a quantitative model (similar to the one of Section 5). This therefore reconciles our results with those of the literature and confirms that the small inflation volatility of Section 5 is not driven by the introduction of the fund nor by the truncation method.

5.5 Numerical robustness

The previous simulations were based on a truncation length of $N = 5$. As explained in Section 4.4.2, we now use LeGrand and Ragot (2022b) to consider heterogeneous truncation lengths. We set the truncation length $N_{\max} = 20$ for the histories with the largest size. The results are provided in Appendix D.7. They are very similar to the results with the uniform truncation method. In particular, the volatility of aggregate variables remain almost unchanged, while the inflation volatility stays very low.

Finally, we also compare the simulation outcomes of the truncation method to those of the histogram method developed by Rios-Rull (1999), Reiter (2009), and Young (2010), which is standard for solving heterogeneous-agent models. We simulate the model calibrated with the previous values, assuming no public spending using the histogram and truncation methods. We then compare their IRFs, their second-order moments, as well as average and maximum absolute differences. The results are reported in Appendix F and appear to be very close.

5.6 Considering constrained-optimal fiscal policy

As a final robustness check, we also simulate the model for a constrained optimal fiscal policy, in which some fiscal tools are fixed in the dynamics, whereas other tools are optimally set. To do so, we assume that the planner considers an aggregate welfare criterion with social weights that are history-specific. This allows us to construct a steady state Ramsey allocation, whose (optimal) fiscal policy is equal to the previous calibration. We can then compute the optimal dynamics around this steady state. More specifically, we consider a general welfare function that places weights on the utility of each agent. For the sake of generality, we assume that these weights are consistent with the sequential representation and depend on initial conditions and an idiosyncratic history. The weight of agent $i \in \mathcal{I}$ at date t is $\omega_{i,t} := \omega_t(y_i^t)$, and the weights satisfy $1 = \int_i \omega_{i,t}\ell(di) = \sum_{y_i^t \in \mathcal{Y}^t} \omega_t(y_i^t)\theta_t(y_i^t)$ for $t \geq 0$. These history-dependent weights are also used by McKay and Wolf (2023) and Dávila and Schaab (2022). Formally, the aggregate welfare criterion can be expressed as follows:

$$W_0 = \mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta^t \int_i \omega_t(y_i^t) U(c_{i,t}, l_{i,t}) \ell(di) \right]. \tag{58}$$

Following the inverse optimal taxation approach, we estimate the social weights such that the observed US fiscal system is optimal at the steady state. We then solve for the optimal inflation and fiscal policy holding the capital tax constant or the labor tax constant. We find that the optimal inflation rate is almost constant, with a standard deviation of 0.01% (on a quarterly basis), confirming the result that the volatility of inflation is low when fiscal policy is partly time-varying. The details of the algebra and of the resolution can be found in Section G.

6 The sticky-wage economy

We now turn to the analysis of the economy with sticky wages, instead of sticky prices. The sticky-wage rigidity is another nominal rigidity studied in the literature, which has both empirical and theoretical supports (see e.g., Chugh, 2006 or Nekarda and Ramey, 2020 and references therein). For the sake of conciseness, we focus here solely on an economy with a mutual fund, constant tax rates and a rule à la Bohn for public debt. This economy is the benchmark economy that is called Economy 1 in Section 5.2, but with sticky wages instead of sticky prices. It corresponds to the environment described in Section 4. The remainder of this section is organized as follows. In Section 6.1, we present the specificities of the sticky-wage environment compared to the previous setup. In Section 6.2, we derive the FOCs of the Ramsey problem in the sticky-wage economy. In Section 6.3, we present the calibration we use. Finally, in Section 6.4, we present the quantitative outcomes of the model and discuss how they compare to those of the sticky-price model.

6.1 The sticky-wage model

Our economy models sticky wages as in the model of wage bargaining of Erceg et al. (2000), which is commonly used in the heterogeneous-agent literature (e.g., Hagedorn et al., 2019, Alves and Violante, 2023 or Auclert et al., 2024). In this model, a continuum of unions of size 1 monopolistically supply labor. These union-specific labor supplies are then pooled together via a competitive aggregator with a constant elasticity of substitution ε_W. Each union k sets its wage W_{kt} so as to maximize the intertemporal welfare of its members. The adjustment of the nominal wage implies a quadratic utility cost that is proportional to the parameter ψ_W, as in Auclert et al. (2024). We focus on the symmetric equilibrium where all unions choose to set the same wage W_t, hence all households work the same number of hours denoted L_t. This construction involves the following wage Phillips curve:

$$\Pi_t^W(\Pi_t^W - 1) = \frac{\varepsilon_W}{\psi_W}\underbrace{\left(v'(L_t) - \frac{\varepsilon_W - 1}{\varepsilon_W} w_t \int_i y_{i,t} u'(c_{i,t})\ell(di)\right)}_{\text{labor gap}} L_t + \beta \mathbb{E}_t\left[\Pi_{t+1}^W(\Pi_{t+1}^W - 1)\right], \quad (59)$$

where $\Pi_t^W = \frac{W_t}{W_{t-1}}$ is the wage inflation, and w_t the real post-tax wage. The details of the derivation of the wage Phillips curve can be found in Appendix I. In equation (59), the *labor gap* term captures the market power of unions that prevents the marginal disutility of labor, $v'(L_t)$, from equalling its marginal benefit. The latter is the sum over all agents of the hourly wage (in efficient units) $w_t y_{i,t}$ multiplied by the marginal utility of consumption $(u'(c_{i,t}))$. Labor supplies of the different unions are imperfect substitutes, which is captured by the term $\frac{\varepsilon_W - 1}{\varepsilon_W}$. When the labor gap is positive, the unions progressively increase wages (due to the wage-adjustment cost), which generates inflationary pressure and contributes to reducing the labor gap. In addition to the labor gap, current wage inflation also depends on expected wage inflation, as is standard in the New-Keynesian literature.

A consequence of this sticky-wage modelling is that agents do not optimally choose their labor supply. The household's budget constraint (45) becomes: $a_{i,t} + c_{i,t} = (1 + r_t)a_{i,t-1} + w_t y_{i,t} L_t + T_t$, where r_t is the fund interest rate that is defined as in Section 4.1 (equations (42)–(44)) The Euler equation (47) is unchanged, but the labor FOC (29) now does not hold. Regarding the production side, firms no longer face quadratic price-adjustment costs. Factor prices now verify the following standard definitions:

$$\tilde{r}_t + \delta = \alpha Z_t(\frac{K_{t-1}}{L_t})^{\alpha-1} \text{ and } \tilde{w}_t = (1-\alpha)Z_t(\frac{K_{t-1}}{L_t})^{\alpha}. \quad (60)$$

6.2 The Ramsey problem

As in the sticky-price economy, the Ramsey planner chooses the monetary-fiscal policy that maximizes the aggregate welfare criterion, while being consistent with individual optimal choices. The aggregate welfare criterion now includes the utility cost of wage inflation:

$$W_0 = \mathbb{E}_0\left[\sum_{t=0}^{\infty}\beta^t \int_i \left(u(c_t^i) - v(L_t)\right)\ell(di) - \frac{\psi_W}{2}(\Pi_t^W - 1)^2\right]. \quad (61)$$

The fiscal policy follows the fiscal rules of Section 4.2. Labor and capital tax rates are constant, while public debt follows the Bohn rule of equation (52). The lump-sum transfers T_t is then determined by the government budget constraint, which still verifies equation (48), but with the price adjustment cost set to zero: $\kappa = 0$. Monetary policy is now composed of three variables: the nominal rate $\tilde{R}_t^N$, wage inflation Π_t^W, and price inflation Π_t^P. Price inflation is a consequence of wage inflation and real wage variations are $\Pi_t^P = \frac{w_{t-1}}{w_t}\Pi_t^W$. Formally, the Ramsey program can be written as follows:

$$\max_{(\Pi_t^P,\Pi_t^W,K_t,w_t,r_t,\tilde{R}_t^N,L_t,(c_{i,t},a_{i,t},\nu_{i,t})_i)_t} W_0, \tag{62}$$

$$G_t + B_{t-1} + r_t(B_{t-1} + K_{t-1}) + w_t L_t + T_t = B_t + Z_t K_{t-1}^{\alpha} L_t^{1-\alpha} - \delta K_{t-1}, \tag{63}$$

$$\text{for all } i \in \mathcal{I}\text{: } a_t^i + c_t^i = (1+r_t)a_{t-1}^i + w_t y_t^i L_t + T_t, \tag{64}$$

$$a_t^i \geq -\bar{a},\ \nu_t^i(a_t^i + \bar{a}) = 0,\ \nu_t^i \geq 0, \tag{65}$$

$$u'(c_t^i) = \beta \mathbb{E}_t\left[(1+r_{t+1})u'(c_{t+1}^i)\right] + \nu_t^i, \tag{66}$$

$$\Pi_t^W(\Pi_t^W - 1) = \frac{\varepsilon_W}{\psi_W}\left(v'(L_t) - \frac{\varepsilon_W - 1}{\varepsilon_W} w_t \int_i y_{i,t} u'(c_{i,t})\ell(di)\right)L_t + \mathbb{E}_t\left[\Pi_{t+1}^W(\Pi_{t+1}^W - 1)\right], \tag{67}$$

$$\Pi_t^P = \frac{w_{t-1}}{w_t}\Pi_t^W \tag{68}$$

together with the post tax-wage definition (9), the fund-related equations (42)–(44), the financial market clearing condition (49), the factor price definitions (60), the fiscal rules (50)–(52), the positivity constraints $c_t^i, l_t^i \geq 0$, and the initial conditions $(a_{-1}^i)_{i\in\mathcal{I}}, B_{-1}$.

The conditions of the program (62)–(68) are straightforward. Equation (73) is the budget of the government, expressed in post-tax terms. Equations (64)–(66) are the individual budget constraints, credit constraints, and individual Euler equations that guarantee that the chosen allocation is consistent with individual optimal choices. All these equations already appear in the Ramsey program with price-rigidity of Section 4.3. Equation (67) is the wage Phillips curve, which replaces the price Phillips curve of the sticky-price Ramsey program. Finally, equation (68) defines the price inflation that appears in the no-profit and no-arbitrage conditions of the fund.

As in the sticky-price economy, it is useful to introduce the concept of the SVL for agent i in a sticky-wage economy, which can be expressed as:

$$\begin{aligned}\psi_t^{SW,i} :=\ & \underbrace{u'(c_t^i)}_{\text{direct effect}} - \underbrace{\left(\lambda_t^i - (1+r_t)\lambda_{t-1}^i\right)u''(c_t^i)}_{\text{effect on savings}} \\ & \underbrace{- \frac{\varepsilon_W - 1}{\psi_W}\gamma_t^W w_t L_t y_{i,t} u''(c_{i,t})}_{\text{effect on wage bargaining}}.\end{aligned} \tag{69}$$

The first two terms ("direct effect" and "effect on savings") in equation (69) are identical to those of the sticky-price expression (see equation (82) in Appendix C.2). Indeed, changing consumption again has an effect on welfare and savings incentives. The main difference between the sticky-price and the sticky-wage cases is the effect on labor supply. In the sticky-price economy, the extra consumption of agent i directly affects their labor supply through the FOC on labor. In the sticky-wage economy, there is no such direct outcome as there is no optimal labor choice. However, the term "effect on wage bargaining" is an indirect outcome on the labor supply set by unions that operates through the labor gap of the wage Phillips curve. This explains why this effect is proportional to the Lagrange multiplier γ^W, which captures the welfare gain for the planner to change wage inflation.[18] The FOCs of the planner can be expressed using $\psi_t^{SW,i}$ and are derived in Appendix I.3 for the sake of conciseness.

We now present a simple implementation result in the sticky-wage economy.[19]

Result 1 (An implementation result) *There exists an allocation implementing zero wage inflation in an economy with exogenous fiscal rules.*

The previous result states the existence of an allocation with zero wage inflation with exogenous fiscal policy. The proof can be found in Appendix I.4. This zero-wage-inflation allocation is appealing. As wages are sticky but prices are not, the planner can reproduce any desired real wage dynamics by changing price inflation through monetary policy, while keeping nominal wages unchanged so as to avoid wage-adjustment cost. The zero-wage inflation would be optimal in the representative-agent economy with sticky wages (Chugh, 2006). In a heterogeneous-agent economy, this zero-wage inflation is not optimal because price inflation involves redistribution across agents, as it affects the nominal interest rate. However, as can be seen in the next section, this redistribution effect is quantitatively small, such that the planner indeed implements an allocation with a low wage inflation volatility and volatile price-inflation.

6.3 The calibration

The calibration strategy is the same as in the sticky-price economy. We here discuss only those parameters that differ from the calibration values listed in Table 1 in Section 1. The fiscal rule is for instance unchanged. The new calibration parameters are reported in Table 3.

Preferences. The scaling parameter is now set to $\chi = 0.061$, so as to again match an aggregate labor supply of roughly $\frac{1}{3}$.

Idiosyncratic risk. We calibrate the productivity process with a persistence of $\rho_y = 0.988$ and a standard deviation of $\sigma_y = 0.154$. We set these values so as to reproduce, as in the sticky-price economy, a steady-state wealth Gini index of 0.73 and an annual capital-to-GDP ratio of 2.5. The resulting values of ρ_y and σ_y are very close to those in the sticky-price economy.[20]

[18] This term does not contradict the fact that agents are atomistic for unions. Indeed, changing the consumption of a unique agent i implies the SVL of equation (69), but does not modify the wage: as the agent is of measure zero, it does not change the term $\int_j y_{j,t} u'(c_{j,t})\ell(dj)$ in the wage Phillips curve (67). However, the planner internalizes the fact that if the consumption of a positive mass of agents is changed, it will affect the wage through the bargaining process of unions.

[19] We could obtain a result similar to Proposition 1: the flexible wage allocation can be recovered when the fiscal system is sufficiently rich. However, since this would significantly lengthen the presentation, we choose to focus here on a result valid for exogenous fiscal rules.

[20] As the labor supply by productivity level is different in the sticky-price and sticky-wage economy, we recalibrate the idiosyncratic productivity process to match the same targets.

Monetary parameters. Following the literature and in particular Schmitt-Grohé and Uribe (2005), we assume that the elasticity of substitution is $\varepsilon_W = 21$ across labor types. The wage adjustment cost is set to $\psi_W = 2100$, to obtain a slope of the wage Phillips curve equal 1%. As in the case of the price Phillips curve, the empirical literature offers a wide range of estimates for the slope of the wage Phillips curve. We show that this does not change our results in Appendix I.5.4. The price adjustment cost is set to $\psi_P = 0$.

Parameter	Description	Value	Target
χ	Scaling param. labor supply	0.061	$L = 1/3$
ρ_y	Autocorrelation idio. income	0.988	Annual capital-to-GDP ratio of 2.5
σ_y	Standard dev. idio. income	15.4%	Gini of wealth of 0.73
κ	Price adjustment cost	0	Slope of the price PC 0%
ε_w	Elasticity of sub. labor inputs	21	Schmitt-Grohé and Uribe (2005)
ψ_w	Wage adjustment cost	700	Slope of the wage PC 3%

Table 3: Parameter values in the baseline calibration. We only report the parameter values which are different from the ones provided in Table 1. See text for descriptions and targets.

As in the sticky-price economy, this calibration generates two untargeted outcomes. The ratio of public debt to annual GDP amounts to 63%, while public spending is equal to 10.7% of annual GDP. These values are slightly different from the ones in the sticky price economy because of the different labor supply.

6.4 Quantitative outcomes

We discuss IRFs and second-order moments in Section 6.4.1, while we conduct the welfare analysis in Section 6.4.2.

6.4.1 Optimal inflation dynamics

We simulate the model following a negative TFP shock of one standard deviation (the same as in Section 5). We also consider three economies, all of which feature the same acyclical fiscal rules of Section 4.2. Economy 1 corresponds to the benchmark economy with optimal monetary policy. Economy 2 features wage stability and hence a constant path for wages: $\Pi_t^W = 1$ at all dates. This allows us to quantify the gains from deviating from wage stability, which is the optimal policy in the representative-agent economy. Economy 3 implements the same Taylor rule as in Section C.2 instead of the optimal inflation path. In Figure 3 we report the IRFs of several key variables for the three economies, which all start from the steady state in period 0. Figure 3 shows that the optimal inflation path is close, but not identical, to nominal wage stability (Panel 7). Obviously, because price inflation has now no direct cost, the real wage adjusts through price inflation, which is now very volatile (Panel 2). The optimal allocation of Economy 1 is

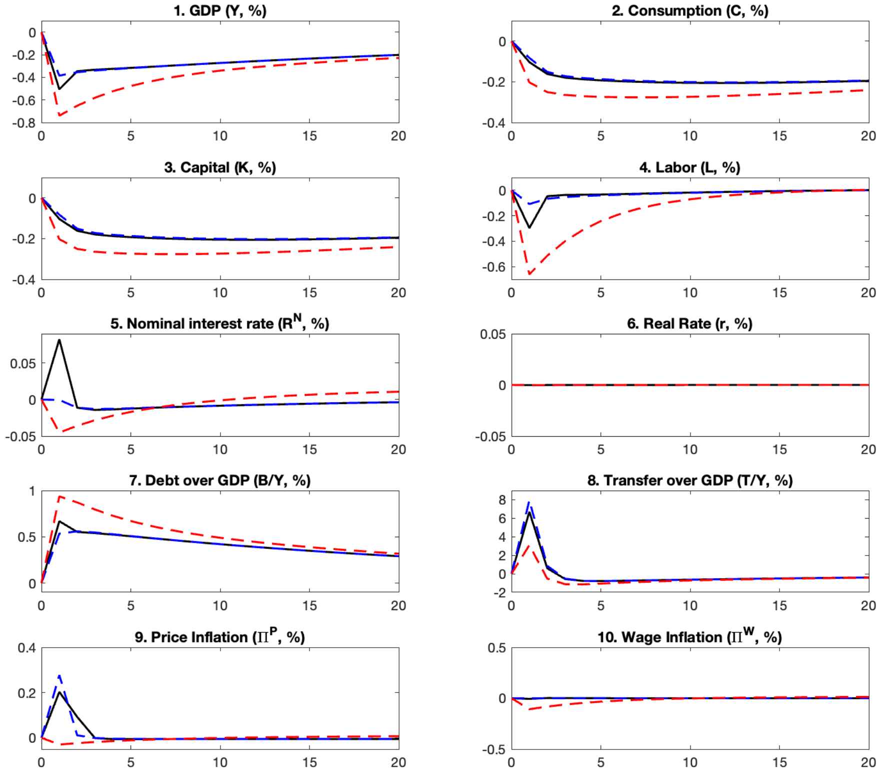

Figure 3: IRFs after a negative productivity shock of one standard deviation for relevant variables. The black line is Economy 1 with optimal monetary policy. The blue dashed line is Economy 2, with constant wage inflation. The red dashed line is Economy 3, where monetary policy follows a Taylor rule.

quantitatively close to, though different from, the allocation of Result 1. The Taylor rule of Economy 3 implies that price inflation markedly differs from the optimal reaction. Consequently, we observe a sizable deviation from optimal allocation.

6.4.2 Welfare comparison

We now report the average welfare gains by productivity associated with implementing optimal policies instead of price stability. We use the same methodology as in Section 5.3, where, as in the price-rigidity case, the benchmark is the optimal monetary-policy case, Economy 1. We use Economy 2 (no wage inflation) to assess the welfare gain of implementing optimal monetary policy. Again, the welfare gaps are measured by the percentage increase in consumption. They are reported in Figure 4 for the different productivity levels. The general shape of the welfare gains per productivity level is overall similar to the one of Figure 2, obtained in the sticky-price economy. The welfare gains are heterogeneous across productivity levels and the lower the productivity level, the higher the welfare gain – even though the average welfare gain is small and of similar magnitude as in the sticky-price case. The latter amounts to 0.001%.

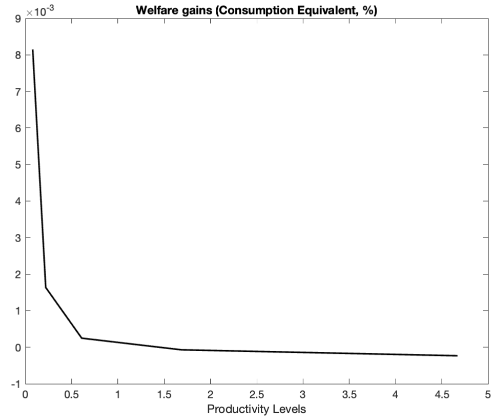

Figure 4: Welfare gaps between Economy 1 (optimal monetary policy) and Economy 2 with no wage inflation ($\Pi^W = 1$). Values are reported as the percentage increase in consumption (y-axis) by productivity level (x-axis).

We have also computed the welfare gaps between the economy with optimal monetary policy and the one with price stability $\Pi^P = 1$. As this last policy is suboptimal even in the representative-agent economy, the welfare gain of optimal policy over fixed price is substantial and amounts to 0.06% on average, which is more than one order of magnitude above the one compared to wage stability.

Finally, we also investigate the robustness of our results to the calibration of the slope of the wage Phillips curve. We report the optimal wage and price inflation responses associated to different values of the slope in Appendix I.5.4. Our main finding is unchanged: wage inflation barely moves, while price inflation reacts sizably to adjust the nominal wage.

7 Conclusion

We derive optimal monetary policy with commitment in an economy with incomplete insurance markets and aggregate shocks, considering sticky prices or sticky wages. Optimal monetary policy is analyzed with different fiscal regimes. When prices are sticky, we first theoretically show that price stability is the optimal policy, when linear capital and labor tax optimally vary other the business cycle. Second, we exogenously set taxes via fiscal rules, which theoretically leaves room for varying inflation. However, for a standard calibration and empirically relevant fiscal rules, the optimal inflation response remains modest. When nominal wages are sticky, price inflation has a new role: it allows the planner to decrease the real wage after a negative supply shock, to bring it closer to the marginal productivity of labor. In this case, we find the welfare gains of deviating from price stability to be sizably high. The welfare gains of deviating from nominal wage stability are lower. In any case, the welfare gains are higher for low productivity workers.

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Appendix

A Computing the FOCs of the full-fledged Ramsey program with flexible prices

This section is organized in two parts. In Section A.1, we transform the Ramsey program by including the Euler equations in the planner's objective. In Section A.2, we derive the FOCs of the Ramsey program in the flexible-price economy.

A.1 Transforming the Ramsey program

We explicitly include the FOCs of the program of the agents (the two Euler equations on consumption and the FOC on labor) in the planner's objective. We recall that the three Lagrange multipliers are denoted by $\beta^t \lambda_{i,k,t}$ for the real Euler equation, $\beta^t \lambda_{i,b,t}$ for the nominal Euler equation, and $\beta^t \lambda_{i,l,t}$ for the labor Euler equation. The objective of the Ramsey program (22)–(31) then becomes:

$$\begin{aligned}
J = & \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \int_i (u(c_{i,t}) - v(l_{i,t}))\ell(di) \\
& - \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \int_i \lambda_{i,k,t} \left(u'(c_{i,t}) - \nu_{i,k,t} - \beta \mathbb{E}_t \left[(1 + r_{t+1}^K) u'(c_{i,t+1}) \right] \right) \ell(di) \\
& - \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \int_i \lambda_{i,b,t} \left(u'(c_{i,t}) - \nu_{i,b,t} - \beta \mathbb{E}_t \left[\Pi_{t+1}^P u'(c_{i,t+1}) \right] \right) \ell(di) \\
& - \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \int_i \lambda_{i,l,t} \left(v'(l_{i,t}) - w_t y_{i,t} u'(c_{i,t}) \right) \ell(di) \\
& - \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \gamma_t \left(\Pi_t^P (\Pi_t^P - 1) Y_t - \frac{\varepsilon - 1}{\kappa} (\zeta_t - 1) Y_t - \beta \mathbb{E}_t \left[\Pi_{t+1}^P (\Pi_{t+1}^P - 1) Y_{t+1} \right] \right).
\end{aligned}$$

In the previous expression, we have used the fact that when the credit constraint is binding for agent i at period t, ($\nu_{i,k,t} > 0$), then there is no Euler equation to consider ($\lambda_{i,k,t} > 0$); in other words: $\nu_{i,k,t}\lambda_{i,k,t} = \nu_{i,b,t}\lambda_{i,b,t} = 0$, due to the complementary slackness conditions. With $\gamma_{-1} = 0$ (which is a harmless normalization), we obtain after some manipulations:

$$\begin{aligned}
J = & \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \int_i (u(c_{i,t}) - v(l_{i,t}))\ell(di) - \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \int_i \left(\lambda_{i,k,t} - (1 + r_t^K) \lambda_{i,k,t-1} \right) u'(c_{i,t}) \ell(di) \qquad (70) \\
& - \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \int_i \left(\lambda_{i,b,t} - \frac{R_t^N}{\Pi_t^P} \lambda_{i,b,t-1} \right) u'(c_{i,t}) \ell(di) - \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \int_i \lambda_{i,l,t} \left(v'(l_{i,t}) - w_t y_{i,t} u'(c_{i,t}) \right) \ell(di) \\
& - \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left((\gamma_t - \gamma_{t-1}) \Pi_t^P (\Pi_t^P - 1) - \frac{\varepsilon - 1}{\kappa} \gamma_t (\zeta_t - 1) \right) Y_t.
\end{aligned}$$

Using (70), the Ramsey program (22)–(31) can now be expressed as: max J on $\big(B_t, T_t, \Pi_t^P, w_t, r_t^K, R_t^N, \tilde{w}_t, \tilde{r}_t^K, \tilde{R}_t^N, K_t, L_t, Y_t, \zeta_t, (c_{i,t}, l_{i,t}, b_{i,t}, k_{i,t}, \nu_{i,t}^b, \nu_{i,t}^k)_i\big)_{t\geq 0}$, subject to the same set of constraints, except the individual Euler equations for consumption (27) and the Phillips curve (30) that have been included in (70). Note that we could also write the Ramsey program using the sequential representation (as in equations (15)), but at the cost of tedious notation.

A.2 FOCs for the flexible-price economy

The flexible-price program implies $\kappa = 0$, and hence $\zeta_t = \Pi_t^P = 1$. Profits are also null. Taking advantage of the post-tax notation, the allocation is the solution to:

$$\max_{\left(B_t, T_t, w_t, r_t^K, R_t^N, (l_{i,t}, b_{i,t}, k_{i,t})_i\right)} J$$

$$c_{i,t} = -k_{i,t} - b_{i,t} + (1 + r_t^K)k_{i,t-1} + R_t^N b_{i,t-1} + w_t y_{i,t} l_{i,t} + T_t,$$

$$G_t + R_t^N B_{t-1} + r_t^K K_{t-1} + w_t L_t + T_t = B_t + Y_t - \delta K_{t-1},$$

and where we use $K_t = \int_i k_{i,t}\ell(di),\ B_t = \int_i b_{i,t}\ell(di),\ L_t = \int_i y_{i,t} l_{i,t}\ell(di),\ Y_t = Z_t K_t^\alpha L_t^{1-\alpha}$, and with complementary slackness conditions.

The Lagrangian can be expressed as follows:

$$\begin{aligned}\mathcal{L} = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \int_i (u(c_{i,t}) - v(l_{i,t}))\ell(di) - \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \int_i \left(\lambda_{i,k,t} - (1 + r_t^K)\lambda_{i,k,t-1}\right) u'(c_{i,t})\ell(di) \\ - \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \int_i \left(\lambda_{i,b,t} - R_t^N \lambda_{i,b,t-1}\right) u'(c_{i,t})\ell(di) - \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \int_i \lambda_{i,l,t} \left(v'(l_{i,t}) - w_t y_{i,t} u'(c_{i,t})\right) \ell(di) \\ + \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \mu_t \left(B_t + Y_t - \delta K_{t-1} - G_t - R_t^N B_{t-1} - r_t^K K_{t-1} - w_t L_t - T_t\right).\end{aligned}$$

Derivative with respect to w_t.

$$0 = \int_i \hat{\psi}_{i,t} y_{i,t} l_{i,t}\ell(di) + \int_i \lambda_{i,l,t} y_{i,t} u'(c_{i,t})\ell(di).$$

Derivative with respect to r_t^K.

$$\int_i \lambda_{i,k,t-1} u'(c_{i,t})\ell(di) + \int_i \hat{\psi}_{i,t} k_{i,t-1}\ell(di) = 0.$$

Derivative with respect to R_t^N. As in the real case, we have:

$$\int_i \lambda_{i,b,t-1} u'(c_{i,t})\ell(di) + \int_i \hat{\psi}_{i,t} b_{i,t-1}\ell(di) = 0.$$

Derivative with respect to $l_{i,t}$.

$$v'(l_{i,t}) + \lambda_{i,l,t} v''(l_{i,t}) = w_t y_{i,t} \left(\hat{\psi}_{i,t} + \mu_t \frac{\tilde{w}_t}{w_t} \right).$$

Derivative with respect to $k_{i,t}$. For unconstrained agents:

$$\begin{aligned} \hat{\psi}_{i,t} &= \beta \mathbb{E}_t \left[(1 + r_{t+1}^K) \hat{\psi}_{i,t+1} \right] + \beta \mathbb{E}_t \left[\left(\tilde{r}_{t+1}^K - r_{t+1}^K \right) \mu_{t+1} \right] + \beta \mathbb{E}_t \left[(1 + r_{t+1}^K) \mu_{t+1} \right] - \mu_t, \\ &= \beta \mathbb{E}_t \left[(1 + r_{t+1}^K) \hat{\psi}_{i,t+1} \right] + \beta \mathbb{E}_t \left[(1 + \tilde{r}_{t+1}^K) \mu_{t+1} \right] - \mu_t \end{aligned}$$

while for constrained ones we have $k_{i,t} = 0$ and $\lambda_{k,i,t} = 0$.

Derivative with respect to $b_{i,t}$. For unconstrained agents:

$$\hat{\psi}_{i,t} = \beta \mathbb{E}_t \left[R_{t+1}^N \hat{\psi}_{i,t+1} \right],$$

while for constrained ones we have $b_{i,t} = 0$ and $\lambda_{b,i,t} = 0$.

Derivative with respect to T_t.

$$\int_i \hat{\psi}_{i,t} \ell(di) \leq 0, \tag{71}$$

with an equality when $T_t > 0$.

B Proof of the equivalence result of Proposition 1

The first step is to simplify the expression of the Ramsey program (22)–(31). Using post-tax notation, we can remove the taxes τ_t^L, τ_t^K, and τ_t^B from the Ramsey program since they can be recovered from post-tax and pre-tax rate w_t, r_t^K, and R^N, as well as $\tilde{w}_t$, $\tilde{r}_t^K$, and $\tilde{R}^N$ using the relationships (9)–(10). We can also remove the pre-tax nominal rate $\tilde{R}_t^N$, as it does not play any role (only the post-tax rate is determined), and can also be recovered from the allocation. The before-tax rates $\tilde{w}_t$ and $\tilde{r}_t^K$ can also be taken out of the Ramsey program, as they can be recovered from the allocation and the markup ζ_t through equations (2) and (5). The profit can also be removed from planner's instruments as it does not directly appear in the program and can be deduced from its definition (8) and the allocation. The second step is to reformulate the Ramsey program by setting $\hat{R}_t = \frac{R_t^N}{\Pi_t^P}$. Since the planner chooses both R_t^N and Π_t^P, this change of variable has no impact on allocation. The path of $(R_t^N)_t$ can be deduced from the paths of $(\Pi_t^P)_t$ and $(\hat{R}_t)_t$.

We can then write the Ramsey program (without further constraints) as follows:

$$\max_{\left(B_t, T_t, \Pi_t^P, w_t, r_t^K, \hat{R}_t, K_t, L_t, Y_t, \zeta_t, (c_{i,t}, l_{i,t}, b_{i,t}, k_{i,t}, \nu_{i,t}^b, \nu_{i,t}^k)_i\right)_{t \geq 0}} W_0, \tag{72}$$

$$\text{s.t. } G_t + \hat{R}_t B_{t-1} + r_t^K K_{t-1} + w_t L_t + T_t = B_t + \left(1 - \frac{\kappa}{2}(\Pi_t^P - 1)^2\right) Y_t - \delta K_{t-1}, \tag{73}$$

$$\text{for all } i \in \mathcal{I}\text{: } c_{i,t} + k_{i,t} + b_{i,t} = (1 + r_t^K) k_{i,t-1} + \hat{R}_t b_{i,t-1} + w_t y_{i,t} l_{i,t} + T_t, \tag{74}$$

$$b_{i,t} \geq -\bar{b}, \nu_{i,t}^b (b_{i,t} + \bar{b}) = 0,\ \nu_{i,t}^b \geq 0, \tag{75}$$

$$k_{i,t} \geq 0, \nu_{i,t}^k k_{i,t} = 0,\ \nu_{i,t}^k \geq 0, \tag{76}$$

$$u'(c_{i,t}) = \beta \mathbb{E}_t \Big[(1 + r_{t+1}^K) u'(c_{i,t+1})\Big] + \nu_{i,t}^k, \tag{77}$$

$$u'(c_{i,t}) = \beta \mathbb{E}_t \Big[\hat{R}_{t+1} u'(c_{i,t+1})\Big] + \nu_{i,t}^b, \tag{78}$$

$$v'(l_{i,t}) = w_t y_{i,t} u'(c_{i,t}), \tag{79}$$

$$B_t = \int_i b_{i,t} \ell(di),\ K_t = \int_i k_{i,t} \ell(di),\ L_t = \int_i y_{i,t} l_{i,t} \ell(di),\ Y_t = Z_t K_{t-1}^{\alpha} L_t^{1-\alpha}, \tag{80}$$

$$\Pi_t^P (\Pi_t^P - 1) = \frac{\varepsilon - 1}{\kappa}(\zeta_t - 1) + \beta \mathbb{E}_t \left[\Pi_{t+1}^P (\Pi_{t+1}^P - 1) \frac{Y_{t+1}}{Y_t}\right]. \tag{81}$$

In the program (72)–(81), the instrument $(\zeta_t)_t$ only plays a role in the Phillips curve (81). The instrument $(\zeta_t)_t$ and the Phillips curve as a constraint can be removed from the Ramsey program and substituted by the constraint $\beta \mathbb{E}_t \left[|\Pi_{t+1}^P (\Pi_{t+1}^P - 1) \frac{Y_{t+1}}{Y_t}|\right] < \infty$, which ensures that the Phillips curve can then used to deduce the path of $(\zeta_t)_t$ from the paths of $(\Pi_t^P)_t$ and $(Y_t)_t$.

It then follows that the Ramsey program simplifies to the set of equations (72)–(80), without $(\zeta_t)_t$ as instrument. Inflation then only appears in the government budget constraint (73) as a cost. This makes it clear that any deviation from price stability (i.e., $\Pi_t^P = 1$ at all dates) shrinks the feasible set of the planner and is hence avoided by the planner. This proves Proposition 1.

To see more precisely that the planner refuses any deviation from price stability, assume that the planner's optimal allocation is $(B_t, T_t, \Pi_t^P, w_t, r_t^K, \hat{R}_t, K_t, L_t, Y_t, (c_{i,t}, l_{i,t}, b_{i,t}, k_{i,t}, \nu_{i,t}^b, \nu_{i,t}^k)_i)_t$ and that there is a date t_0 in which the planner chooses to deviate from price stability: $\Pi_{t_0}^P \neq 1$. Consider the allocation $(B_t, T_t', \Pi_t^{P\prime}, w_t, r_t^K, \hat{R}_t, K_t, L_t, Y_t, \zeta_t, (c_{i,t}', l_{i,t}, b_{i,t}, k_{i,t}, \nu_{i,t}^b, \nu_{i,t}^k)_i)_t$ that is identical to the initial one, but for the transfer, inflation and individual consumption. We define for all $t \neq t_0$: $T_t' = T_t$, $\Pi_t^{P\prime} = \Pi_t^P$, and $c_{i,t}' = c_{i,t}$ and for t_0: $T_{t_0}' = T_{t_0} + \frac{\kappa}{2}(\Pi_{t_0}^P - 1)^2 Y_{t_0}$, $\Pi_{t_0}^{P\prime} = 1$, and $c_{i,t_0}' = c_{i,t} + \frac{\kappa}{2}(\Pi_{t_0}^P - 1)^2 Y_{t_0} > c_{i,t}$. In words, the prime allocation is the same as the non-prime one, except at date t_0, where gross inflation has been modified from $\Pi_{t_0}^P \neq 1$ to 1 and the output destruction $\frac{\kappa}{2}(\Pi_{t_0}^P - 1)^2 Y_{t_0}$ that has been avoided has been transferred to agents via the lump-sum instrument. It is easy to check that the prime allocation is feasible (i.e., that it verifies all constraints (73)–(80)) and that it implies a strictly better aggregate welfare (labor supply is unchanged, consumption at date t_0 is strictly higher and consumption at other dates unchanged). This contradicts that the non-prime allocation is optimal and shows that the planner never chooses to deviate from price stability when all fiscal instruments are available.

C Suboptimal fiscal policy

In this section, we characterize the optimal monetary policy when the fiscal policy follows ad-hoc rules (Section C.2). We start with estimating the empirically relevant rules (Section C.1).

C.1 Empirical characterization of the tax rate cyclicality.

We characterize here the empirically relevant fiscal rules. We extend the results of Vegh and Vuletin (2015) and show that labor and capital taxes can indeed be considered to be acyclical in the United States. We consider fiscal data that covers the period 1960-2023.[21] We report in Table 4 some descriptive statistics about the changes in personal and corporate tax rates in the US for the period 1960-2023. The first three columns reports the numbers of changes. More precisely, if $(\tau_t)_{t=0,\dots,T}$ is our sample of tax rates, a tax change observed at date $t = 1, \dots, T$ is $\tau_t - \tau_{t-1} \neq 0$. It is an increase if the difference is positive and a decrease if the difference is negative. The first two columns thus report the number of such dates in our sample. The last two columns report the average value of the tax variations $\frac{\tau_t - \tau_{t-1}}{\tau_{t-1}}$, either over all dates t or only over all dates t for which there is a tax change.[22] The personal tax rate changes 12 times over the period, which implies that, on average, the tax rate is changed approximately every five years. Similarly, the corporate income tax changes 10 times over the period, which implies that, on average, the tax rate is changed every 6 years.

Tax rates	Number of changes			Value of the change	
	Increase	Decrease	Total	Mean	Mean when $\neq 0$
Personal	3	9	12	-1.11	-5.85
Corporate	2	8	10	-0.94	-5.93

Table 4: Descriptive statistics about the changes in personal and corporate tax rates in the United-States in 1960-2023.

To assess to which extent tax rates vary with the business cycle, we compute the average percentage change in tax rates in booms and recessions. We follow Vegh and Vuletin (2015) and define three stances of the business cycle as a function of the quartile of the real GDP cycles, computed using the Hodrick–Prescott filter. Good times correspond to the years in which the real GDP cycles belong to the top quartile. Conversely, bad times are the years in which the real GDP cycles belong to the bottom quartile. Other years with the real GDP cycles in the middle two quartiles represent normal times. We then compute the percentage changes for personal

[21] The updated dataset is available on Guillermo Vuletin's website: `https://www.guillermovuletin.com/datasets`. The result holds for both the highest and the average marginal personal income tax rates but we present here results for the highest marginal personal income tax rate.

[22] Formally, these two averages correspond to $\frac{1}{T-1}\sum_{t=2}^{T}\frac{\tau_t-\tau_{t-1}}{\tau_{t-1}}$ and $\frac{1}{\mathrm{Card}(T_{change})}\sum_{t\in T_{change}}\frac{\tau_t-\tau_{t-1}}{\tau_{t-1}}$,where $T_{change} = \{t \in \{2, \dots, T\} : \tau_t - \tau_{t-1} \neq 0\}$.

and corporate tax rates in the three stances of the business cycle, with respect to the overall mean percentage change in the sample. Formally, this corresponds to the demeaned average percentage change in tax rates $\left(\frac{\tau_t - \tau_{t-1}}{\tau_{t-1}} - \overline{\delta\tau}\right) \times 100$, where $\overline{\delta\tau}$ is the overall mean tax percentage change and the date t is restricted to belong to one of the business cycle stance. We report these values in Table 5. A positive value indicates tax rates that are higher than the average, while negative values indicate that rates are lower than the average. We can observe that personal and corporate tax rates are increasing in both good and bad times. This is indicative that these two tax policies are acyclical.

	Personal	Corporate
Good times	1.03	1.30
Normal times	−1.85	−1.26
Bad times	2.55	1.12

Table 5: Percentage tax rate changes across different stances of the business cycle. Percentage tax rate changes are expressed as differences from the overall average tax rate.

We also compute the coefficient of correlation between the tax changes and the real GDP growth rate. More precisely, we compute the coefficient of correlation between $(\frac{\tau_t - \tau_{t-1}}{\tau_{t-1}})_{t=2,\ldots,T}$ and $(\frac{GDP_t}{GDP_{t-1}})_{t=2,\ldots,T}$. We report in Table 6 these correlations and their p-values. The correlations are very small, close to zero and not significantly different from 0. This confirms our previous diagnostic that tax rates are acyclical.

	Real GDP	p-values
Personal	−0.025	0.85
Corporate	−0.020	0.90

Table 6: Correlations between tax changes and real GDP.

C.2 Program with suboptimal fiscal policy

Because pre- and post-tax rates cannot be set independently, nominal frictions cannot be suppressed in this setup. We have two additional Lagrange multipliers: $\beta^t \Upsilon_t$ on the no-arbitrage condition (43), and $\beta^t \Gamma_t$ on the zero-profit condition (42) of the fund. The Lagrange multiplier on the unique Euler equation is denoted $\beta^t \lambda_t$. Because of exogenous fiscal rules, the planner's instruments are: $a_{i,t}$, $l_{i,t}$, for individual variables and $\tilde{R}_t^N$, r_t, Π_t^P and ζ_t for aggregate variables. Because agents only save in a fund, we also need to redefine the SVL of equation (32) as:

$$\psi_{i,t} := \underbrace{u'(c_{i,t})}_{\text{direct effet}} - \underbrace{(\lambda_{i,t} - (1+r_t)\lambda_{i,t-1})\, u''(c_{i,t})}_{\text{effect on savings}} + \underbrace{\lambda_{i,l,t} y_{i,t} w_t u''(c_{i,t})}_{\text{effect on labor supply}}. \tag{82}$$

Derivative with respect to $\tilde{R}_t^N$.

$$\mathbb{E}_t\left[(1-\tau_{t+1}^K)\frac{\Gamma_{t+1}}{\Pi_{t+1}^P}\right]B_t = \beta^{-1}\Upsilon_t\mathbb{E}_t\left[\frac{1}{\Pi_{t+1}^P}\right]. \tag{83}$$

Derivative with respect to ζ_t.

$$\begin{aligned}0 &= w_t\int_i y_{i,t}\left(\psi_{i,t}l_{i,t}+\lambda_{i,l,t}u'(c_{i,t})\right)\ell(di) - w_t\mu_t L_t \\ &+ \frac{\varepsilon-1}{\kappa}\zeta_t\gamma_t Y_t - (\tilde{r}_t^K+\delta)\left(\beta^{-1}\Upsilon_{t-1}+\Gamma_t(1-\tau_t^K)K_{t-1}\right).\end{aligned} \tag{84}$$

Derivative with respect to Π_t^P.

$$0 = \mu_t\kappa\left(\Pi_t^P-1\right)+(\gamma_t-\gamma_{t-1})(2\Pi_t^P-1)+\left(\beta^{-1}\Upsilon_{t-1}-\Gamma_t(1-\tau_t^K)B_{t-1}\right)\frac{\tilde{R}_{t-1}^N}{Y_t\left(\Pi_t^P\right)^2}. \tag{85}$$

Derivative with respect to r_t.

$$0 = \int_i\left(\psi_{i,t}a_{i,t-1}+\lambda_{i,t-1}u'(c_{i,t})\right)\ell(di)+(\Gamma_t-\mu_t)A_{t-1}. \tag{86}$$

Derivative with respect to $a_{i,t}$. For unconstrained agents:

$$\begin{aligned}\psi_{i,t} &= \beta\mathbb{E}_t\left[(1+r_{t+1})\psi_{i,t+1}\right] \\ &+ \beta\frac{\alpha}{K_t}\mathbb{E}_t\left[w_{t+1}\int_j y_{j,t+1}\left(\psi_{j,t+1}l_{j,t+1}+\lambda_{l,j,t+1}u'(c_{j,t+1})\right)\ell(dj)\right] \\ &- \beta\frac{\alpha}{K_t}\mathbb{E}_t\left[\left((\gamma_{t+1}-\gamma_t)\Pi_{t+1}^P(\Pi_{t+1}^P-1)-\frac{\varepsilon-1}{\kappa}\gamma_{t+1}(\zeta_{t+1}-1)\right)Y_{t+1}\right] \\ &+ \beta\frac{\alpha}{K_t}\mathbb{E}_t\left[\mu_{t+1}\left(Y_{t+1}\left(1-\frac{\kappa}{2}(\Pi_{t+1}^P-1)^2\right)-\frac{1}{\alpha}(r_{t+1}+\delta)K_t-w_{t+1}L_{t+1}\right)\right] \\ &- \beta\frac{\alpha-1}{K_t}\mathbb{E}_t\left[(\tilde{r}_{t+1}^K+\delta)\left(\beta^{-1}\Upsilon_t+\Gamma_{t+1}(1-\tau_{t+1}^K)K_t\right)\right] \\ &+ \beta\mathbb{E}_t\left[\Gamma_{t+1}(r_{t+1}-(1-\tau_{t+1}^K)\tilde{r}_{t+1}^K)\right],\end{aligned} \tag{87}$$

while for constrained ones we have $a_{i,t} = -\bar{a}$ and $\lambda_{i,t} = 0$.

Derivative with respect to T_t.

$$\mu_t \leq \int_j \psi_{j,t}\ell(dj),$$

with equality if $T_t > 0$.

Derivative with respect to $l_{i,t}$.

$$
\begin{aligned}
w_t \int_i \psi_t^i y_t^i \ell(di) = {}& \alpha w_t \int_i \psi_t^i y_t^i \ell(di) + v'(L_t) \\
& - \frac{\varepsilon_W}{\psi_W}\gamma_t^W \left(v'(L_t) + L_t v''(L_t) - \frac{\varepsilon_W - 1}{\varepsilon_W}(1-\alpha) w_t \int_i y_{i,t} u'(c_{i,t})\ell(di) \right) \\
& + (1-\alpha)\frac{\mu_t}{L_t}\left[w_t L_t - Y_t \right] \\
& + \left[\beta^{-1}\Upsilon_{t-1} + (1-\tau_t^K)\Gamma_t K_{t-1} \right](1-\alpha)\frac{\tilde{r}_t^K + \delta}{L_t} \\
& + \left(\beta E_t\left[\Lambda_{t+1}\Pi_{t+1}^W \right] - \Pi_t^P \Lambda_t \right)\alpha \frac{w_t}{L_t}
\end{aligned}
$$

D The truncated representation

We derive the planner's FOCs for truncated economies.

D.1 The aggregation procedure

As explained in Section 2.6, the sequential representation characterizes the equilibrium of the full-fledged model by sequences of functions depending on the full aggregate and idiosyncratic histories of agents. Our aggregation procedure involves expressing the model based on so-called *truncated histories*, which are N-length vectors $h := (y_{-N+1}, \ldots, y_0)$ representing agents' idiosyncratic histories over the last N periods. The quantity $N \geq 0$ is the truncation length. Loosely speaking, we can represent the truncated history of an agent i whose idiosyncratic history is y^t as:

$$
y^t = \{\ldots, y_{-N-2}^h, y_{-N-1}^h, y_{-N}^h, \underbrace{y_{-N+1}^h, \ldots, y_{-1}^h, y_0^h}_{=h}\}, \tag{88}
$$

where h is the truncated history, corresponding to the idiosyncratic history over the last N periods. We now turn to the different steps of the aggregation procedure. We denote by $\mathcal{H} := \mathcal{Y}^N$ the set of all truncated histories.

First, the measure of agents with truncated history h, denoted by S_h, can be computed as:

$$
S_h = \sum_{h \in \mathcal{H}} S_{\hat{h}} \pi_{\hat{h}h}, \tag{89}
$$

where $\pi_{\hat{h}h}$ is the probability to switch from truncated history $\hat{h}$ at $t-1$ to truncated history h at t. It is equal to:

$$
\pi_{\hat{h}h} = 1_{h \succeq \hat{h}} \pi_{y_0^{\hat{h}} y_0^h}, \tag{90}
$$

and thus to the probability to switch from current productivity level $y_0^{\hat{h}}$ to current state y_0^h if h is a possible continuation of $\hat{h}$ (denoted by $h \succeq \hat{h}$).

Second, the model aggregation implies to assign consumption, saving, and labor choices to groups of agents with the same truncated history. For the sake of simplicity, we will write *truncated history* for "the group of agents sharing the same truncated history". Take the case of a generic variable, denoted by $X_t(y^t, z^t)$, where we make the dependence in y^t and z^t explicit. The quantity assigned to truncated history $h \in \mathcal{H}$ is denoted by $X_{t,h}$ and equal to the average value of X among agents with truncated history h. Formally:

$$X_{t,h}(z^t) = \frac{1}{S_{t,h}} \sum_{y^t \in \mathcal{Y}^t | (y^t_{t-N+1},\ldots,y^t_t)=h} X_t(y^t,z^t)\theta_t(y^t), \tag{91}$$

where $\theta_t(y^t)$ is recalled to be the measure of agents with history y^t. Definition (91) can be applied to the average consumption, the end-of-period saving, the labor supply, and the credit-constraint Lagrange multiplier respectively and lead to the quantities $c_{t,h}$, $a_{t,h}$, $l_{t,h}$, and $\nu_{t,h}$.

Third, we compute the aggregate beginning-of-period wealth. Applying definition (91) to date-t beginning-of-period wealth requires to account that agents transit from one truncated history at $t-1$ to another at t. The beginning-of-period wealth $\tilde{a}_{t,h}$ for truncated history h is:

$$\tilde{a}_{t,h} = \sum_{\hat{h} \in \mathcal{H}} \frac{S_{t-1,\hat{h}}}{S_{t,h}} \pi_{\hat{h}h} a_{t-1,\hat{h}}. \tag{92}$$

Fourth, the aggregation of the different equations characterizing the equilibrium is rather straightforward except for Euler equations – which involve non-linear marginal utilities. Indeed, the marginal utility of consumption aggregation ($u'(c_{t,h})$) is different from the aggregation of marginal utility ($u'_{t,h}$): $u'(c_{t,h}) \neq u'_{t,h}$. The ratio of these two scalars will be denoted by $\xi_{t,h}$, which guarantees that Euler equations hold with aggregate consumption levels. Similarly, we denote by $\xi_{l,t,h}$ the parameters associated to the labor Euler equation.

D.2 The truncated model

We can now proceed with the aggregation of the full-fledged model. First, the aggregation of individual budget constraints (45) using equations (91) and (92) yields the following equation:

$$a_{t,h} + c_{t,h} = w_t y_0^h l_{t,h} + (1 + r_t)\tilde{a}_{t,h} + T_t, \text{ for } h \in \mathcal{H}. \tag{93}$$

The aggregation of Euler equations for consumption (16) and labor (18) implies for any $h \in \mathcal{H}$:

$$\xi_h u'(c_{t,h}) = \beta \mathbb{E}_t \left[(1 + r_{t+1}) \sum_{\tilde{h} \succeq h} \pi_{h\tilde{h}} \xi_{\tilde{h}} u'(c_{t+1,\tilde{h}}) \right] + \nu_{t,h}, \tag{94}$$

$$\xi_{l,h} v'(l_{t,h}) = w_t y_0^h \xi_h u'(c_{t,h}). \tag{95}$$

The system consisting of equations (93)–(95) is an exact aggregation of the full-fledged model with aggregate shocks in terms of truncated idiosyncratic histories. This system characterizes the dynamics of the aggregated variables $c_{t,h}$, $a_{t,h}$, $l_{t,h}$ and $\nu_{t,h}$ without any approximation.

Finally, we express market clearing conditions (19) using aggregated variables:

$$K_t = \sum_{h\in\mathcal{H}} S_{t,h}a_{t,h}, \quad L_t = \sum_{h\in\mathcal{H}} S_{t,h}y_0^h l_{t,h}. \tag{96}$$

The parameters ξs that appear in the aggregated Euler equations (94) are key in our method. We both show how to compute them using steady-state allocations and how these computations can be used to simulate the model in the presence of aggregate shocks.

Steady state and computation of the ξs. At the steady state, the computations of the parameters ξs can be done based on allocations. Indeed, we can compute the stationary wealth distribution of the full-fledged model (using the individual equations) and identify credit-constrained histories. We can then compute aggregate (steady-state) allocations, c_h, a_h, l_h and ν_h, using equations (91) and (92). Then, consumption Euler equations (94) and (95) can be inverted to compute the parameters $(\xi_h)_h$ and $(\xi_{l,h})_h$ capturing the within-history heterogeneity in consumption and labor. Actually, this computation involves only standard linear algebra, and we provide a closed-form expression for the ξs – see equations (110) and (111) in Appendix D.5.

The truncated model in the presence of aggregate shocks. To use our truncation method in the presence of aggregate shocks, two further assumptions are needed.

Assumption A *We make the following two assumptions.*

1. *The parameters $(\xi_h)_h$ and $(\xi_{l,h})_h$ remain constant and equal to their steady-state values.*

2. *The set of credit-constrained histories, denoted by $\mathcal{C} \subset \mathcal{H}$, is time-invariant.*

The resulting model (i.e., the aggregated model with Assumption A) is called the truncated model. We therefore use the ξs twice: (i) once exactly to estimate them using the steady-state allocation; (ii) once approximately to simulate the model in the presence of aggregate shocks.[23]

Finally, two properties are worth mentioning. First, by construction of the ξs, the allocations of the full-fledged Bewley model and of the truncated model coincide with each other at the steady state. Second, the allocations of the truncated model (in the presence of aggregate shocks) can be proved to converge to those of the full-fledged equilibrium (and the ξs to 1), when the truncation length N becomes increasingly long (see LeGrand and Ragot, 2022a). Furthermore, from a quantitative perspective, Section 5 shows that the ξs are an efficient tool to capture the heterogeneity within truncated histories, even when the truncation length is not too large.

[23]Assuming that the ξs are constant in the dynamics is equivalent to assuming that the distribution of agents within the truncated history is constant. This has the same spirit as assuming that the distribution within bins of wealth is uniform in the histogram method of Reiter (2009), among others.

D.3 Ramsey program

Thanks to its finite state-space representation, the truncated model makes it possible to solve the Ramsey program in the presence of aggregate shocks, which is a challenging task. The Ramsey program in the truncated economy can be expressed as follows.

$$\max_{(w_t,r_t,\tilde{r}_t^K,\tilde{R}_t^N,\tau_t^K,\tau_t^L,B_t,T_t,K_t,L_t,\Pi_t^P,\zeta_t,(a_{t,h},c_{t,h},l_{t,h})_h)_{t\geq 0}} \mathbb{E}_0\left[\sum_{t=0}^{\infty}\beta^t\sum_{h\in\mathcal{H}}S_h\xi_h(u(c_{t,h})-v(l_{t,h}))\right], \tag{97}$$

subject to aggregate Euler equations (94) and (95), aggregate budget constraint (93), aggregate market clearing conditions (96), credit constraints $a_{t,h}\geq -\bar{a}$, as well as constraints that were already present in the full-fledged Ramsey program: the governmental budget constraint (23), the Phillips curve (30), the definition (2) of ζ_t, the one (4) of Y_t, the ones (9) and (10) of after-tax rates r_t, r_t^K, R_t^N and w_t, the zero profit condition for the fund (42), the no-arbitrage constraint (43), and the relationship (5) between factor prices.

As in Section 2.7, it is possible to use the tools of Marcet and Marimon (2019) to rewrite the Ramsey program. The truncation adds no complexity to the formulation of the planner's objective. FOCs can similarly be derived as in the general case, and we obviously have the same equivalence results.[24]

D.4 Program in the economy with sub-optimal fiscal policy

D.4.1 Program formulation

We take advantage of the equivalence result to simplify the program expression, which is:

$$\max_{(\tilde{R}_t^N,r_t,\zeta_t,B_t,(a_{t,h},c_{t,h},l_{t,h})_{h\in\mathcal{H}})_{t\geq 0}} \mathbb{E}_0\sum_{t=0}^{\infty}\beta^t\sum_{h\in\mathcal{H}}\Big[S_{t,h}\Big(u(c_{t,h})-v(l_{t,h}) \tag{98}$$

$$-\left(\lambda_{t,h}-\tilde{\lambda}_{t,h}(1+r_t)\right)\xi_h u'(c_{t,h})\Big)-\lambda_{l,t,h}\left(\xi_{l,h}v'(l_{t,h})-w_t y_0^h\xi_h u'(c_{t,h})\right)\Big)\Big],$$

$$\text{s.t. } c_{t,h}+a_{t,h}=w_t y_0^h l_{t,h}+(1+r_t)\tilde{a}_{t,h}+T_t, \tag{99}$$

$$a_{t,h}\geq 0 \text{ and } \tilde{a}_{t,h}=\sum_{\tilde{y}^N\in\mathcal{Y}^N}\pi_{\tilde{h}h}\frac{S_{t-1,\tilde{h}}}{S_{t,h}}a_{t-1,\tilde{h}}, \tag{100}$$

$$\tilde{\lambda}_{t,h}=\frac{1}{S_{t,h}}\sum_{\tilde{h}\in\mathcal{H}}S_{t-1,\tilde{h}}\lambda_{t-1,\tilde{h}}\pi_{\tilde{h}h}, \tag{101}$$

$$K_t=\sum_h S_{t,h}a_{t,h}-B_t,\ L_t=\sum_h S_{t,h}y_h^0 l_{t,h}, \tag{102}$$

[24] A final aspect regarding the truncated Ramsey program is that its solutions can be shown to converge to the solutions of the full-fledged Ramsey program (if they exist), when the truncation length N becomes infinitely long. See LeGrand and Ragot (2022a, Proposition 5). This convergence property is the parallel of the convergence result regarding allocations of the competitive equilibrium.

and subject to other constraints that are the same as in the individual case (government budget constraint, Phillips curve, fund no-arbitrage condition, fund zero profit condition, the definitions of ζ and of profits).

D.4.2 FOCs in the truncated model

We define the net SVL for history h similarly to definition (32):

$$\psi_{t,h} = \xi_h u'(c_{t,h}) - \Big(\lambda_{t,h} - \tilde{\lambda}_{t,h}(1+r_t)\Big)\, \xi_h u''(c_{t,h}) + \lambda_{l,h} y_0^h w_t u''(c_{t,h}). \tag{103}$$

The FOCs with respect to $\tilde{R}_t^N$, and Π_t^P are unchanged compared to the individual case.

Derivative with respect to r_t.

$$0 = \sum_{h\in\mathcal{H}} S_{t,h}\psi_{t,h}\tilde{a}_{t,h} + \sum_{h\in\mathcal{H}} S_{t,h}\tilde{\lambda}_{t,h}\xi_h u'(c_{t,h}) + (\Gamma_t - \mu_t)A_{t-1}. \tag{104}$$

Derivative with respect to ζ_t.

$$\begin{aligned} 0 = {} & w_t \sum_{h\in\mathcal{H}} S_{t,h} y_0^h \left(l_{t,h}\psi_{t,h} + \lambda_{l,t,h} u'(c_{t,h})\right) - w_t\mu_t L_t \\ & + \frac{\varepsilon-1}{\kappa}\zeta_t\gamma_t Y_t - (\tilde{r}_t^K + \delta)\left(\beta^{-1}\Upsilon_{t-1} + \Gamma_t(1-\tau_t^K)K_{t-1}\right). \end{aligned}$$

Derivative with respect to $a_{t,h}$. For unconstrained truncated histories (i.e., $\nu_h = 0$):

$$\begin{aligned} \psi_{t,h} = {} & \beta\mathbb{E}_t\Big[(1+r_{t+1})\sum_{\tilde{h}\in\mathcal{H}} \pi_{h\tilde{h}}\hat{\psi}_{t+1,\tilde{h}}\Big] + \beta\mathbb{E}_t\left[\Gamma_{t+1}(r_{t+1} - (1-\tau_{t+1}^K)\tilde{r}_{t+1}^K)\right] \\ & + \beta\frac{\alpha}{K_t}\mathbb{E}_t\left[w_{t+1}\sum_{h\in\mathcal{H}} S_{t,h} y_0^h \left(l_{t+1,h}\psi_{t+1,h} + \lambda_{l,t+1,h}u'(c_{t+1,h})\right)\ell(dj)\right] \\ & - \beta\frac{\alpha}{K_t}\mathbb{E}_t\left[\left((\gamma_{t+1}-\gamma_t)\Pi_{t+1}^P(\Pi_{t+1}^P - 1) - \frac{\varepsilon-1}{\kappa}\gamma_{t+1}(\zeta_{t+1}-1)\right)Y_{t+1}\right] \\ & + \beta\frac{\alpha}{K_t}\mathbb{E}_t\left[\mu_{t+1}\left(Y_{t+1}\left(1 - \frac{\kappa}{2}(\Pi_{t+1}^P - 1)^2\right) - \frac{1}{\alpha}(r_{t+1}+\delta)K_t - w_{t+1}L_{t+1}\right)\right] \\ & - \beta\frac{\alpha-1}{K_t}\mathbb{E}_t\left[(\tilde{r}_{t+1}^K + \delta)\left(\beta^{-1}\Upsilon_t + \Gamma_{t+1}(1-\tau_{t+1}^K)K_t\right)\right]. \end{aligned} \tag{105}$$

For constrained agents, we have $a_{t,h} + \bar{a} = 0$ and $\lambda_{t,h} = 0$.

Derivative with respect to T_t.

$$\sum_h S_{t,h}\psi_{t,h} \le \mu_t, \tag{106}$$

with equality when $T_t > 0$.

Derivative with respect to $l_{t,h}$.

$$\begin{aligned}\psi_{t,h} = &\frac{1}{w_t y_0^h} v'(l_{t,h}) + \frac{\lambda_{l,t,h}}{w_t y_0^h} v''(l_{t,h}) + \alpha \frac{1}{L_t} \sum_{h \in \mathcal{H}} S_{t,h} y_0^h \left(l_{t,h}\psi_{t,h} + \lambda_{l,t,h} u'(c_{t,h})\right) \\ &+ \frac{1-\alpha}{w_t L_t}\left(\mu_t w_t L_t + (\tilde{r}_t^K + \delta)\left(\beta^{-1}\Upsilon_{t-1} + \Gamma_t(1-\tau_t^K)K_{t-1}\right)\right) \\ &+ \frac{1-\alpha}{w_t L_t}\left((\gamma_t - \gamma_{t-1})\Pi_t^P(\Pi_t^P - 1) - \frac{\varepsilon - 1}{\kappa}\gamma_t(\zeta_t - 1) - \mu_t\left(1 - \frac{\kappa}{2}(\Pi_t^P - 1)^2\right)\right) Y_t.\end{aligned} \tag{107}$$

D.5 Matrix representation of the steady-state allocation

We provide here closed-form formulas for the parameters ξs and the Lagrange multipliers, so as to solve the steady-state model.

As a preliminary, note that at the steady state, we have $Z = 1$, and the FOCs (83) and (85) (with respect to $\tilde{R}_t^N$ and Π_t^P) imply that $\Pi^P = 1$, and using the Phillips curve, $\zeta = 1$.[25]

The matrix representation consists in stacking together the equations characterizing the steady-state, so as to provide a convenient matrix notation for solving the steady state. It also provides an efficient solution to compute the values for the ξs and the Lagrange multiplier. We recall that there are N_{tot} truncated histories and we assume that the set of truncated histories is endowed with a total order that is used for indexing vectors and matrices. In other words, $(x_h)_h$ implicitly assumes that all elements x_h are collected along a given order that remains identical from one vector to another.

Let $\boldsymbol{S}$ be the N_{tot}-vector of steady-state history sizes that is defined as $\boldsymbol{S} = (S_h)_h$. Similarly, let $\boldsymbol{a}$, $\boldsymbol{c}$, and $\boldsymbol{\nu}$ be the N_{tot}-vectors of end-of-period wealth, consumption, credit-constraint Lagrange multipliers, respectively. These vectors are known from the steady-state equilibrium of the Bewley model. Denote by $\boldsymbol{I}$ the $(N_{tot} \times N_{tot})$-identity matrix, $\boldsymbol{1}_{N_{tot}}$ the $(N_{tot} \times 1)$-vector of 1, and $\boldsymbol{\Pi}$ as the transition matrix across histories. Denoting by $\circ$ the Hadamard product and by $\times$ or an absence of mathematical sign the usual matrix product, we obtain the following steady-state relationships:

$$\boldsymbol{S} = \boldsymbol{\Pi}^\top \boldsymbol{S}, \tag{108}$$

$$\boldsymbol{S} \circ \boldsymbol{c} + \boldsymbol{S} \circ \boldsymbol{a} = (1+r)\boldsymbol{\Pi}^\top(\boldsymbol{S} \circ \boldsymbol{a}) + \boldsymbol{S} \circ \boldsymbol{W} \circ \boldsymbol{l}, \tag{109}$$

where (108) is the dynamics of history sizes and (109) is the history of budget constraints.

Euler equations (94) and (95) imply that we are looking for vectors $\boldsymbol{\xi}^u$ and $\boldsymbol{\xi}^v$ such that:

$$\boldsymbol{\xi}^u \circ u'(\boldsymbol{c}) = \beta(1+r)\boldsymbol{\Pi}\left(\boldsymbol{\xi}^u \circ u'(\boldsymbol{c})\right) + \boldsymbol{\nu}, \tag{110}$$

$$\boldsymbol{\xi}^v \circ v'(\boldsymbol{l}) = w\boldsymbol{\xi}^u \circ \boldsymbol{y} \circ u'(\boldsymbol{c}), \tag{111}$$

where the matrix $\boldsymbol{\Pi}^\top$ (the transpose of $\boldsymbol{\Pi}$) is used to make expectations about next-period

[25]We remove the t subscript to denote steady-state quantities.

histories and where $u'(\boldsymbol{c})$ assumes that the function applies element-wise (and thus denotes $(u'(c_h))_h$). Finally, for any vector $\boldsymbol{x}$, we denote by $\boldsymbol{D_x}$ the diagonal matrix with $\boldsymbol{x}$ on the diagonal. We deduce from (110) that $\boldsymbol{\xi}^u$ is defined by the following closed-form formula:

$$u'(\boldsymbol{c}) \circ \boldsymbol{\xi}^u = (\boldsymbol{I} - \beta(1+r)\boldsymbol{\Pi})^{-1}\boldsymbol{\nu}. \tag{112}$$

We then use $\boldsymbol{\xi}^v \circ v'(\boldsymbol{l}) = w\boldsymbol{\xi}^u \circ \boldsymbol{y} \circ u'(\boldsymbol{c})$ to compute $\boldsymbol{\xi}^v$.

We set an initial guess value for μ. We start with the definitions of the $\tilde{\lambda}_c$s and of the ψs of equations (101) and (103):

$$\boldsymbol{S} \circ \tilde{\boldsymbol{\lambda}}_c = \boldsymbol{\Pi}^\top(\boldsymbol{S} \circ \boldsymbol{\lambda}_c), \tag{113}$$

$$\boldsymbol{\psi} = \boldsymbol{\xi}^u \circ u'(\boldsymbol{c}) - \boldsymbol{D}_{\boldsymbol{\xi}^u \circ u''(\boldsymbol{c})}\left(\boldsymbol{\lambda}_c - (1+r)\tilde{\boldsymbol{\lambda}}_c - w\boldsymbol{y} \circ \boldsymbol{\lambda}_l\right). \tag{114}$$

Combining these two equations yields:

$$\boldsymbol{S} \circ \boldsymbol{\psi} = \boldsymbol{S} \circ \boldsymbol{\xi}^u \circ u'(\boldsymbol{c}) - \boldsymbol{D}_{\boldsymbol{\xi}^u \circ u''(\boldsymbol{c})}\left(\boldsymbol{I} - (1+r)\boldsymbol{\Pi}^\top\right)(\boldsymbol{S} \circ \boldsymbol{\lambda}_c) + \boldsymbol{D}_{w\boldsymbol{y} \circ \boldsymbol{\xi}^u \circ u''(\boldsymbol{c})}(\boldsymbol{S} \circ \boldsymbol{\lambda}_l), \tag{115}$$

where $\circ$ is commutative and for any vectors $\boldsymbol{x}$ and $\boldsymbol{y}$: $\boldsymbol{x} \circ \boldsymbol{y} = \boldsymbol{D_x y}$ and $\boldsymbol{D_x D_y} = \boldsymbol{D}_{\boldsymbol{x} \circ \boldsymbol{y}}$.

We also denote: $\tilde{\xi}^v_h = \frac{\xi^v_h}{y^h_0}$. The FOC (107) for labor becomes:

$$\begin{aligned}
\boldsymbol{S} \circ \boldsymbol{\psi} &= \frac{1}{w}\boldsymbol{S} \circ \tilde{\boldsymbol{\xi}}^v \circ v'(\boldsymbol{l}) + \frac{1}{w}\tilde{\boldsymbol{\xi}}^v \circ v''(\boldsymbol{l}) \circ (\boldsymbol{S} \circ \boldsymbol{\lambda}_l) \\
&\quad + (\alpha - 1)\frac{r + (1-\tau^K)\delta}{wL}\left(\boldsymbol{S}\tilde{\boldsymbol{a}}^\top(\boldsymbol{S} \circ \boldsymbol{\psi}) + \boldsymbol{S}(\boldsymbol{\xi}^u \circ u'(\boldsymbol{c}))^\top(\boldsymbol{S} \circ \tilde{\boldsymbol{\lambda}}_c)\right) \\
&\quad + \alpha\frac{1}{L}\left(\boldsymbol{S}(\boldsymbol{y} \circ \boldsymbol{l})^\top(\boldsymbol{S} \circ \boldsymbol{\psi}) + \boldsymbol{S}(\boldsymbol{y} \circ \boldsymbol{\xi}^u \circ u'(\boldsymbol{c}))^\top(\boldsymbol{S} \circ \tilde{\boldsymbol{\lambda}}_l)\right) \\
&\quad - \frac{\mu}{wL}(1-\alpha)\left(Y - (r + (1-\tau^K)\delta)A - wL\right)\boldsymbol{S}.
\end{aligned}$$

Denoting $\boldsymbol{M}_0 = wL\boldsymbol{I} - (\alpha-1)(r + (1-\tau^K)\delta)\boldsymbol{S}\tilde{\boldsymbol{a}}^\top - \alpha w\boldsymbol{S}(\boldsymbol{y} \circ \boldsymbol{l})^\top$, we have:

$$\begin{aligned}
\boldsymbol{M}_0\boldsymbol{S} \circ \boldsymbol{\psi} &= (\alpha-1)(r + (1-\tau^K)\delta)(\boldsymbol{S}(\boldsymbol{\xi}^u \circ u'(\boldsymbol{c}))^\top\boldsymbol{\Pi}^\top)(\boldsymbol{S} \circ \boldsymbol{\lambda}_c) \\
&\quad + (\boldsymbol{D}_{L\tilde{\boldsymbol{\xi}}^v \circ v''(\boldsymbol{l})} + \alpha w(\boldsymbol{S}(\boldsymbol{y} \circ \boldsymbol{\xi}^u \circ u'(\boldsymbol{c}))^\top\boldsymbol{\Pi}^\top))(\boldsymbol{S} \circ \boldsymbol{\lambda}_l) \\
&\quad + L\boldsymbol{S} \circ \boldsymbol{\omega} \circ \tilde{\boldsymbol{\xi}}^v \circ v'(\boldsymbol{l}) - \mu(1-\alpha)\left(Y - (r + (1-\tau^K)\delta)A - wL\right)\boldsymbol{S},
\end{aligned} \tag{116}$$

where it should be observed that $\boldsymbol{S}(\boldsymbol{\xi}^u \circ u'(\boldsymbol{c}))^\top\boldsymbol{\Pi}^\top$ has the dimension $(N_{tot} \times 1) \cdot (1 \times N_{tot}) \cdot (N_{tot} \times N_{tot}) = (N_{tot} \times N_{tot})$ and is a matrix. Similarly, $\boldsymbol{S}(\boldsymbol{\xi}^u \circ u'(\boldsymbol{c}) \circ \boldsymbol{y})^\top\boldsymbol{\Pi}^\top$, $\boldsymbol{S}\tilde{\boldsymbol{a}}^\top$, and $\boldsymbol{S}(\boldsymbol{y} \circ \boldsymbol{l})^\top$ also have the dimension $(N_{tot} \times 1) \cdot (1 \times N_{tot}) = (N_{tot} \times N_{tot})$ and are matrices.

Using (115): $\boldsymbol{S} \circ \boldsymbol{\psi} = \boldsymbol{\omega} \circ \boldsymbol{S} \circ \boldsymbol{\xi}^u \circ u'(\boldsymbol{c}) - \boldsymbol{D}_{\boldsymbol{\xi}^u \circ u''(\boldsymbol{c})}\left(\boldsymbol{I} - (1+r)\boldsymbol{\Pi}^\top\right)(\boldsymbol{S} \circ \boldsymbol{\lambda}_c) + \boldsymbol{D}_{w\boldsymbol{y} \circ \boldsymbol{\xi}^u \circ u''(\boldsymbol{c})}(\boldsymbol{S} \circ \boldsymbol{\lambda}_l)$, we deduce:

$$\boldsymbol{S} \circ \boldsymbol{\lambda}_c = \boldsymbol{L}_0(\boldsymbol{S} \circ \boldsymbol{\lambda}_l) + \boldsymbol{x}_0, \tag{117}$$

$$\text{where: } \tilde{\boldsymbol{L}}_0 = \boldsymbol{M}_0 \boldsymbol{D}_{\boldsymbol{\xi}^u \circ u''(\boldsymbol{c})} \left(\boldsymbol{I} - (1+r)\boldsymbol{\Pi}^\top \right) + (\alpha - 1)(r + (1-\tau^K)\delta)\boldsymbol{S}(\boldsymbol{\xi}^u \circ u'(\boldsymbol{c}))^\top \boldsymbol{\Pi}^\top,$$
$$\boldsymbol{L}_0 = \tilde{\boldsymbol{L}}_0^{-1} (\boldsymbol{M}_0 \boldsymbol{D}_{w\boldsymbol{y} \circ \boldsymbol{\xi}^u \circ u''(\boldsymbol{c})} - \boldsymbol{D}_{L\tilde{\boldsymbol{\xi}}^v \circ v''(\boldsymbol{l})} - \alpha w (\boldsymbol{S}(\boldsymbol{y} \circ \boldsymbol{\xi}^u \circ u'(\boldsymbol{c}))^\top \boldsymbol{\Pi}^\top)),$$
$$\boldsymbol{x}_0 = \tilde{\boldsymbol{L}}_0^{-1} \Big(\boldsymbol{M}_0 \boldsymbol{\omega} \circ \boldsymbol{S} \circ \boldsymbol{\xi}^u \circ u'(\boldsymbol{c}) - L\boldsymbol{S} \circ \boldsymbol{\omega} \circ \tilde{\boldsymbol{\xi}}^v \circ v'(\boldsymbol{l})$$
$$+ \mu(1-\alpha) \left(Y - (r + (1-\tau^K)\delta)A - wL \right) \boldsymbol{S} \Big).$$

We now turn to the Euler-like equation (105) that holds only for unconstrained histories. We introduce the matrix $\boldsymbol{\Pi}^\psi$ defined for any vector $\boldsymbol{x}$ as:

$$\boldsymbol{\Pi}^\psi (\boldsymbol{S} \circ \boldsymbol{x}) = \boldsymbol{S} \circ (\boldsymbol{\Pi} \boldsymbol{x}), \tag{118}$$

and the matrix $\boldsymbol{P}$, which is the matrix with one on the diagonal if the history is unconstrained and 0 otherwise. With this notation, equation (105) becomes:

$$\begin{aligned} \boldsymbol{P}(\boldsymbol{S} \circ \boldsymbol{\psi}) = {} & \beta(1+r)\boldsymbol{P}\boldsymbol{\Pi}^\psi(\boldsymbol{S} \circ \boldsymbol{\psi}) \\ & + \frac{\beta}{K}(\alpha - 1)(r + (1-\tau^K)\delta)\boldsymbol{P}\left(\boldsymbol{S}\tilde{\boldsymbol{a}}^\top(\boldsymbol{S} \circ \boldsymbol{\psi}) + \boldsymbol{S}(\boldsymbol{\xi}^u \circ u'(\boldsymbol{c}))^\top(\boldsymbol{S} \circ \tilde{\boldsymbol{\lambda}}_c) \right) \\ & + \frac{\beta}{K}\alpha w \boldsymbol{P}\left(\boldsymbol{S}(\boldsymbol{y} \circ \boldsymbol{l})^\top(\boldsymbol{S} \circ \boldsymbol{\psi}) + \boldsymbol{S}(\boldsymbol{y} \circ \boldsymbol{\xi}^u \circ u'(\boldsymbol{c}))^\top(\boldsymbol{S} \circ \tilde{\boldsymbol{\lambda}}_l) \right) \\ & + \frac{\beta}{K}\boldsymbol{P}\mu\left(\alpha Y - (\alpha - 1)(r + (1-\tau^K)\delta)A - (r+\delta)K - \alpha w L \right)\boldsymbol{S}. \end{aligned} \tag{119}$$

Denoting: $\tilde{\boldsymbol{L}}_1 = \boldsymbol{I} - \beta(1+r)\boldsymbol{\Pi}^\psi - \frac{\beta}{K}(\alpha-1)(r+(1-\tau^K)\delta)\boldsymbol{S}\tilde{\boldsymbol{a}}^\top - \frac{\beta}{K}\alpha w \boldsymbol{S}(\boldsymbol{y} \circ \boldsymbol{l})^\top$, equation (119) becomes:

$$\begin{aligned} \boldsymbol{P}\tilde{\boldsymbol{L}}_1(\boldsymbol{S} \circ \boldsymbol{\psi}) = {} & \frac{\beta}{K}(\alpha - 1)(r + (1-\tau^K)\delta)\boldsymbol{P}\boldsymbol{S}(\boldsymbol{\xi}^u \circ u'(\boldsymbol{c}))^\top \boldsymbol{\Pi}^\top (\boldsymbol{S} \circ \boldsymbol{\lambda}_c) \\ & + \frac{\beta}{K}\alpha w \boldsymbol{P}\boldsymbol{S}(\boldsymbol{y} \circ \boldsymbol{\xi}^u \circ u'(\boldsymbol{c}))^\top \boldsymbol{\Pi}^\top (\boldsymbol{S} \circ \boldsymbol{\lambda}_l) \\ & + \frac{\beta}{K}\boldsymbol{P}\mu\left(\alpha Y - (\alpha - 1)(r + (1-\tau^K)\delta)A - (r+\delta)K - \alpha w L \right)\boldsymbol{S}. \end{aligned}$$

Using (115) to express $\boldsymbol{S} \circ \boldsymbol{\psi}$, we obtain:

$$\boldsymbol{P}\boldsymbol{L}_{1,c}(\boldsymbol{S} \circ \boldsymbol{\lambda}_c) = \boldsymbol{P}\boldsymbol{L}_{1,l}(\boldsymbol{S} \circ \boldsymbol{\lambda}_l) + \boldsymbol{P}\boldsymbol{x}_1, \tag{120}$$
$$\text{where: } \tilde{\boldsymbol{L}}_1 = \boldsymbol{I} - \beta(1+r)\boldsymbol{\Pi}^\psi - \frac{\beta}{K}(\alpha-1)(r+(1-\tau^K)\delta)\boldsymbol{S}\tilde{\boldsymbol{a}}^\top - \frac{\beta}{K}\alpha w \boldsymbol{S}(\boldsymbol{y} \circ \boldsymbol{l})^\top,$$
$$\boldsymbol{L}_{1,c} = \tilde{\boldsymbol{L}}_1 \boldsymbol{D}_{\boldsymbol{\xi}^u \circ u''(\boldsymbol{c})} \left(\boldsymbol{I} - (1+r)\boldsymbol{\Pi}^\top \right) + \frac{\beta}{K}(\alpha - 1)(r+\delta)\boldsymbol{S}(\boldsymbol{\xi}^u \circ u'(\boldsymbol{c}))^\top \boldsymbol{\Pi}^\top,$$
$$\boldsymbol{L}_{1,l} = \tilde{\boldsymbol{L}}_1 \boldsymbol{D}_{w\boldsymbol{y} \circ \boldsymbol{\xi}^u \circ u''(\boldsymbol{c})} - \frac{\beta}{K}\alpha w \boldsymbol{S}(\boldsymbol{y} \circ \boldsymbol{\xi}^u \circ u'(\boldsymbol{c}))^\top \boldsymbol{\Pi}^\top,$$
$$\boldsymbol{x}_1 = \tilde{\boldsymbol{L}}_1 \boldsymbol{\omega} \circ \boldsymbol{S} \circ \boldsymbol{\xi}^u \circ u'(\boldsymbol{c})$$
$$- \frac{\beta}{K}\mu\left(\alpha Y - (\alpha - 1)(r + (1-\tau^K)\delta)A - (r+\delta)K - \alpha w L \right)\boldsymbol{S}.$$

We also have for unconstrained agents: $(\boldsymbol{I}-\boldsymbol{P})(\boldsymbol{S}\circ\boldsymbol{\lambda}_c)=0$, which yields with (120):

$$\begin{aligned}\boldsymbol{S}\circ\boldsymbol{\lambda}_c &= \boldsymbol{L}_2(\boldsymbol{S}\circ\boldsymbol{\lambda}_l)+\boldsymbol{x}_2, \\ \text{where: } \boldsymbol{L}_2 &= (\boldsymbol{I}-\boldsymbol{P}+\boldsymbol{P}\boldsymbol{L}_{1,c})^{-1}\boldsymbol{P}\boldsymbol{L}_{1,l}, \\ \boldsymbol{x}_2 &= (\boldsymbol{I}-\boldsymbol{P}+\boldsymbol{P}\boldsymbol{L}_{1,c})^{-1}\boldsymbol{P}\boldsymbol{x}_1.\end{aligned} \tag{121}$$

Combining (117) and (121), we obtain:

$$\begin{aligned}\boldsymbol{S}\circ\boldsymbol{\lambda}_l &= -(\boldsymbol{L}_0-\boldsymbol{L}_2)^{-1}(\boldsymbol{x}_0-\boldsymbol{x}_2), \\ \boldsymbol{S}\circ\boldsymbol{\lambda}_c &= \boldsymbol{x}_2-\boldsymbol{L}_2(\boldsymbol{L}_2-\boldsymbol{L}_0)^{-1}(\boldsymbol{x}_2-\boldsymbol{x}_0),\end{aligned}$$

and we deduce:

$$\begin{aligned}\boldsymbol{S}\circ\boldsymbol{\psi} = \boldsymbol{S}\circ\boldsymbol{\xi}^u\circ u'(\boldsymbol{c}) - \boldsymbol{D}_{\boldsymbol{\xi}^u\circ u''(\boldsymbol{c})}\left(\boldsymbol{I}-(1+r)\boldsymbol{\Pi}^\top\right)\boldsymbol{x}_2 + \boldsymbol{D}_{w\boldsymbol{y}\circ\boldsymbol{\xi}^u\circ u''(\boldsymbol{c})}(\boldsymbol{S}\circ\boldsymbol{\lambda}_l), \\ + (\boldsymbol{D}_{\boldsymbol{\xi}^u\circ u''(\boldsymbol{c})}\left(\boldsymbol{I}-(1+r)\boldsymbol{\Pi}^\top\right) - \boldsymbol{D}_{w\boldsymbol{y}\circ\boldsymbol{\xi}^u\circ u''(\boldsymbol{c})})(\boldsymbol{L}_0-\boldsymbol{L}_2)^{-1}(\boldsymbol{x}_0-\boldsymbol{x}_2).\end{aligned} \tag{122}$$

It remains to iterate on μ until FOC (106) holds, or: $\mu=\boldsymbol{1}^\top(\boldsymbol{S}\circ\boldsymbol{\psi})$.

We then compute the steady-state value of other Lagrange multipliers using the FOCs (83), (84), and (104):

$$\begin{aligned}\Gamma &= \mu - \frac{1}{\boldsymbol{S}^\top\boldsymbol{a}}\left(\tilde{\boldsymbol{a}}^\top(\boldsymbol{S}\circ\boldsymbol{\psi}) + (\boldsymbol{\xi}^u\circ u'(\boldsymbol{c}))^\top(\boldsymbol{S}\circ\boldsymbol{\lambda}_c)\right), \\ \Upsilon &= \beta(1-\tau^K)\Gamma B, \\ \frac{\varepsilon-1}{\kappa}\gamma Y &= (\tilde{r}^K+\delta)\left(\beta^{-1}\Upsilon+\Gamma_t(1-\tau^K)K\right)+w\mu L \\ &\quad - w(\boldsymbol{y}\circ\boldsymbol{l})^\top(\boldsymbol{S}\circ\boldsymbol{\psi}) - w(\boldsymbol{y}\circ u'(\boldsymbol{c}))^\top(\boldsymbol{S}\circ\boldsymbol{\lambda}_l).\end{aligned}$$

D.6 Second-order moments

The second-order moments of the three economies discussed in Section 5.2 are presented in Table 7. We simulate the economy considering the TFP process given in Table 1. We report the unconditional first- and second-order moments for the main variables, in the three economies. For each variable, we report the steady-state value (labeled "Mean") and the normalized standard deviation in percent, equal to the standard deviation divided by the mean (labeled "Normalized std deviation"), except for taxes and inflation, for which the standard deviation is reported. The second part of the table reports correlations. We can observe that Table 7 confirms the IRFs regarding aggregate variables. The volatility of aggregate consumption is slightly lower in Economy 1 than in Economies 2 and 3.

Variables	Mean	Normalized std deviation (%) Economy 1	Economy 2	Economy 3
Y	1.431	1.480	1.481	1.539
C	0.900	1.338	1.343	1.373
K	14.311	1.547	1.548	1.635
L	0.392	0.221	0.174	0.234
$\tilde{R}^N$	1.011	0.054	0.033	0.101
r	0.007	0.025	0.022	0.025
B	3.637	0.619	0.619	0.613
T	0.114	4.705	6.045	5.427
Π^P	1.000	0.020	0.000	0.067
Π^W	1.000	0.449	0.285	0.359
Correlations				
$corr(\Pi^P, Y)$		0.204	0.00	0.113
$corr(\Pi^W, Y)$		0.125	0.157	0.157
$corr(B, Y)$		−0.971	−0.972	−0.976
$corr(C, Y)$		0.949	0.943	0.934
$corr(Y, Y_{-1})$		0.979	0.967	0.967
$corr(B, B_{-1})$		0.960	0.966	0.966

Table 7: First- and second-order moments for key variables in the three economies (Economy 1 is the benchmark; Economy 2 is the economy with constant inflation; Economy 3 is the economy with the Taylor rule).

D.7 A robustness check: The refined truncation

D.7.1 The refinement method

The refinement method, developed in LeGrand and Ragot (2022b) for 2 idiosyncratic states, consists in considering truncated histories of unequal length, instead of truncated histories of the same length N, as in Section D. The idea is that some histories that are particularly large or heterogeneous would benefit from longer length. For example, with two idiosyncratic states (corresponding to high productivity y_2 and to low productivity y_1), the set of truncated histories of length 2 is $\{(y_2, y_2), (y_2, y_1), (y_1, y_2), (y_1, y_1)\}$, where (y_2, y_2) and (y_1, y_1) are typically the two largest truncated histories, since productivity is persistent. In that case, the refinement method allows one to split these larger histories, without affecting the smaller histories (y_2, y_1), and (y_1, y_2). For instance, the history (y_2, y_2) can be refined into (y_2, y_2, y_2) and (y_1, y_2, y_2), such that the set of truncated histories is now $\{(y_2, y_2, y_2), (y_1, y_2, y_2), (y_2, y_1), (y_1, y_2), (y_1, y_1)\}$. It can be checked that: (i) this set corresponds to a proper partition of idiosyncratic histories (every infinite history admits a unique truncated representation in the set) and (ii) the transition matrix between set histories is well-defined.[26]

We extent the refinement method of LeGrand and Ragot (2022b) to $k := Card(\mathcal{Y})$ idiosyncratic states. We denote a set of refined truncated histories by $R(N, (N_i)_{i=1,\dots,k})$, where N is the uniform truncation length (on which the refinement is based), and $N_i \geq N$ is the longest refinement history for the state i. The construction is recursive and follows the same steps as the above example. It starts from the set $R(N, (N)_{i=1,\dots,k})$ of uniform truncated histories that are all of length N. The first refinement step consists in substituting for the history $y_1^N = (\underbrace{y_1, \dots, y_1}_{N})$, the set of histories $\{(y_i, \underbrace{y_1, \dots, y_1}_{N}) : i = 1, \dots, k\}$. This yields the set $R(N, N+1, (N_i)_{i=2,\dots,k})$ of size $k^N + k - 1$. The second step consists in refining the history $y_1^{N+1} = (\underbrace{y_1, \dots, y_1}_{N+1})$ into $\{(y_i, \underbrace{y_1, \dots, y_1}_{N+1}) : i = 1, \dots, k\}$, which gives the set $R(N, N+2, (N_i)_{i=2,\dots,k})$, which is of size $k^N + 2(k-1)$. These steps are repeated until we obtain $R(N, N_1, (N_i)_{i=2,\dots,k})$, which is of size $k^N + (N_1 - N)(k-1)$. Furthermore, this set implies a well-defined mapping between infinite and truncated histories, as well as a well-defined transition probability matrix. The same method can be applied to any other history of the form $y_i^N = (\underbrace{y_i, \dots, y_i}_{N})$ with $i = 2, \dots, k$. This finally generates the set $R(N, (N_i)_{i=1,\dots,k})$, with $k^N + (k-1)\sum_{i=1}^{k}(N_i - N)$ elements.

The truncation method can then be used as in the uniform case of Section D. Equations characterizing the Ramsey allocation in Section D.4 and the matrix representation of Section D.5 remain valid once the proper transition matrix has been computed (see equation (90)).

D.7.2 Quantitative application to the model of Section 5

We consider a refined truncation based on a uniform truncation with $N = 5$. We then consider a common refinement length of 20 periods for all constant histories (hence, a set $R(5, (20)_{i=1,\dots,7})$ with the notation of the previous section). The number of truncated histories with positive size increases to 907 from 727 with the uniform truncation method. We report in Table 8 the second-order moments associated to the simulation of Economies 1 and 2 of Section 5.2 using the refined truncation method. As can be seen from the comparison with Table 7, the differences with the uniform truncation method are small, of the order of magnitude of 10^{-4}. This is due to the better accuracy of the refined truncation method. In any case, the results on the volatility of both price and wage inflation are quantitatively similar.

[26] The last point does not hold in general. Indeed, $\{(y_2, y_2), (y_2, y_1, y_2), (y_1, y_1, y_2), (y_1)\}$ corresponds to the refinement of $\{(y_2, y_2), (y_1, y_2), (y_1)\}$ (through (y_1, y_2)) but does not yield a well-defined transition matrix.

Variables	Mean	Normalized std deviation (%) Economy 1	Economy 2
Y	1.431	1.499	1.499
C	0.900	1.340	1.343
K	14.311	1.600	1.599
L	0.392	0.223	0.180
$\tilde{R}^N$	1.011	0.054	0.034
r	0.007	0.025	0.023
B	3.637	0.619	0.62
T	0.114	4.785	6.079
Π^P	1.00	0.020	0.000
Π^W	1.00	0.426	0.285
Correlations			
$corr(\Pi^P, Y)$		0.206	0.000
$corr(\Pi^W, Y)$		0.127	0.154
$corr(B, Y)$		−0.971	−0.971
$corr(C, Y)$		0.945	0.94
$corr(Y, Y_{-1})$		0.979	0.968
$corr(B, B_{-1})$		0.960	0.966

Table 8: Refined truncation (20 periods). First- and second-order moments for key variables, in the two economies (Economy 1 features acyclical fiscal rule and optimal monetary policy; Economy 2 features acyclical fiscal rule and constant inflation).

D.8 A robustness check: Cyclical tax rates

We investigate the robustness of our quantitative results of Section 5 to a modification of our fiscal rules. We relax the assumption of constant marginal rate and assume that our fiscal policy is determined by the rule of equation (52), and by the two following rules:

$$\tau_t^L = \tau_*^L - \sigma_1^L(Z_t - Z_*) - \sigma_2^L(Z_{t-1} - Z_*), \tag{123}$$

$$\tau_t^K = \tau_*^K - \sigma_1^K(Z_t - Z_*) - \sigma_2^K(Z_{t-1} - Z_*), \tag{124}$$

instead of the rules of equations (50)–(51). We calibrate this fiscal rule by setting the six parameters $(\sigma_1^L, \sigma_2^L, \sigma_1^K, \sigma_2^K, \sigma^T, \sigma^B) = (-0.6, 0.5, 0.6, -0.5, 8.5, 4.0)$. The calibration is such that labor taxes fall in recessions, whereas capital tax increases. This rule implements a similar debt path to our benchmark rule, because the reduction on labor tax is compensated by an increase in capital taxes. However, compared to our benchmark acyclical rule, this rule implies different insurance properties against the aggregate risk. We call the economy with this cyclical fiscal rule and optimal monetary policy "Economy 4".

As in Section 5 of the main text, we simulate the model after a negative TFP shock of one

Variables	Mean	Normalized std deviation (%) Economy 1	Economy 4
Y	1.431	1.480	1.443
C	0.900	1.338	1.310
K	14.311	1.547	1.501
L	0.392	0.221	0.194
$\tilde{R}^N$	1.011	0.054	0.051
r	0.007	0.025	0.024
B	3.637	0.619	0.621
T	0.114	4.705	5.191
Π^P	1.000	0.020	0.019
Π^W	1.000	0.449	0.433
Correlations			
$corr(\Pi^P, Y)$		0.204	0.215
$corr(\Pi^W, Y)$		0.125	0.124
$corr(B, Y)$		-0.971	-0.972
$corr(C, Y)$		0.949	0.949
$corr(Y, Y_{-1})$		0.979	0.980
$corr(B, B_{-1})$		0.960	0.961
$corr(\tau^L, Y)$		0.000	0.507
$corr(\tau^K, Y)$		0.000	-0.507

Table 9: First- and second-order moments for key variables, in Economy 1 (acyclical fiscal policy) and Economy 4 (time-varying fiscal policy).

standard deviation. We compare the outcomes of Economies 1 and 4. As a reminder, Economy 1 is defined in Section 5.2 and features acyclical fiscal policy and optimal monetary policy. Second-order moments are reported in Table 9. We do not report the IRFs, which are very similar in the two economies. Second-order moments show that a mildly counter-cyclical tax system contributes to reduce the volatility of optimal inflation and of other aggregate variables and thus that time-varying fiscal policy provides some insurance against aggregate shocks.

E Relationship with the literature: Theoretical and quantitative investigations

E.1 Role of the slope of the Phillips curve

We verify that our results are robust to reasonable variations in the slope of the Phillips curve. Indeed, if there is a strong consensus about the positivity of the slope, empirical estimates are less conclusive about its precise value. To address potential concerns regarding the influence of this value on our result, we consider a wide variety of values for the slope that correspond to the various estimates that can be found in the literature. More precisely, we consider the following values, with between brackets the corresponding reference: 0.62% (Hazell et al., 2022), 1.9% (Rotemberg and Woodford, 1997), 5% (our benchmark), 8.5% (Galí, 2015), 23% (Barnichon and Geert, 2020). In the Calvo models, these slopes imply the following average price durations, in quarters: 14.0, 8.0, 5.1, 4.0, and 2.7. These different values of the slope then imply different values for the cost of inflation κ, since the slope in the Rotemberg model is given by $(\varepsilon-1)/\kappa$ and since we consider the same elasticity $\varepsilon = 6$ as in the baseline case. We simulate the model with these different values of κ, while keeping the rest of the calibration identical to the benchmark one, given in Table 1. In Figure 5, we report quarterly wage and price inflation (in %) after a negative TFP shock (the y-axis of the two plots have different scales). We find that price inflation increases at most to 0.04% after a shock. On the opposite, wage inflation varies sizably and allows the real wage to get closer to the marginal productivity of labor.

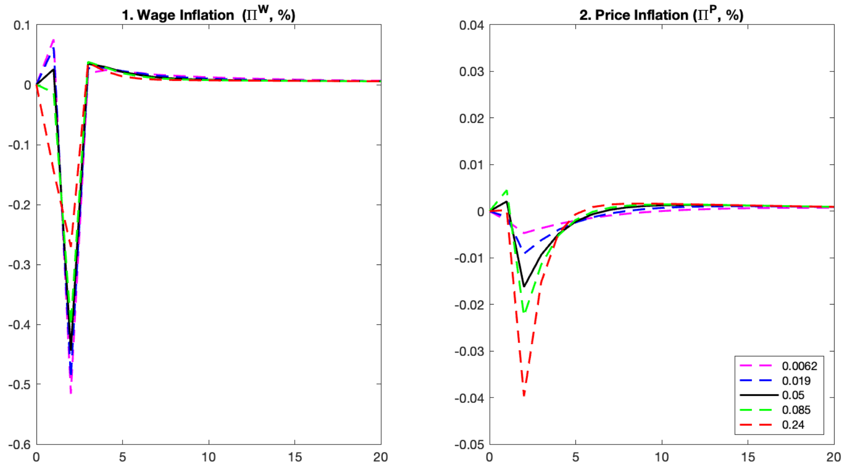

Figure 5: IRFs in percentage points after a negative productivity shock. Quarterly wage inflation (left-hand side) and price inflation (right-hand side) are given in percentage point. The different curves correspond to different values of the slope of the Phillips curve given in the legend.

E.2 A two-agent economy

We consider a simplified version of the quantitative model of Section 4, with a modification of the way profits are distributed. We remove capital and assume that there are two productivity levels $y_2 > y_1 > 0$ with an equal mass 1/2 of agents at each productivity level. Agents are also assumed to inelastically supply one unit of labor and to trade only nominal assets. Productivity levels are assumed to be chosen for the aggregate labor supply to be equal to 1: $\frac{y_1+y_2}{2} = 1$ and the transition matrix (π_{ij}) is symmetric. There is no government (hence, no public spending, no tax, and no public debt). Nominal assets are thus in zero-net supply.

The firms' profits, Ω_t, are not taxed, but are distributed to households as a function of their productivity. More precisely, agents of type i receive a share of profits Ω_t equal to $(\frac{y_1^\nu+y_2^\nu}{2})^{-1}y_i^\nu$, where $\nu \geq 0$ characterizes how unequal the profit distribution is (note that shares sum to 1). When $\nu = 0$, all agents receive the same share of total profits, and the higher ν, the more the profit distribution is tilted toward high-productivity agents. At the limit ($\nu \to \infty$), all profits are distributed to the highest-productivity agents only.[27]

Furthermore, this simple economy is characterized by a specific risk-sharing arrangement, which corresponds to the "island" metaphor.[28] Agents with the same productivity level pool resources and consume the same amount, as if they were on the same island. Agents move across the two islands (corresponding to the two productivity levels) according to their productivity status. Formally, this economy is similar to a truncated model in which the truncation length is set equal to one and the ξs are all set to one. The equilibrium has indeed a simple structure with two agents (with high and low productivity), which allows us to easily derive the optimal policy with standard methods.

E.2.1 The model without capital

Because of the risk sharing arrangement, the after-pooling wealth on island i at date t, denoted $\tilde{a}_{i,t}$, can be expressed as function of the end-of-period wealth, $a_{i,t}$, as $\tilde{a}_{1,t} = \pi_{11}a_{1,t} + \pi_{21}a_{2,t}$ and $\tilde{a}_{2,t} = \pi_{12}a_{1,t} + \pi_{22}a_{2,t}$. Using a guess-and-verify strategy, we assume that type-1 agents are credit-constrained: $a_{1,t} = -\bar{a}$ at all dates. We will later provide the conditions for this to be the case. As assets are in zero net-supply, the end-of-period market clearing is: $a_{1,t} + a_{2,t} = 0$. We deduce $a_{1,t} = -a_{2,t} = -\bar{a}$ and $\tilde{a}_{1,t} = -\tilde{a}_{2,t} = -(\pi_{11} - \pi_{21})\bar{a}$. In the absence of capital ($\alpha = 0$) and because of the normalization of the labor supply, the production sector is $Y_t = Z_t$, and we deduce from Section 2.3 the following expression for profits: $\Omega_t = (1 - \frac{w_t}{Z_t} - \frac{\kappa}{2}(\Pi_t^P - 1)^2)Z_t L_t$, where we used $\zeta_t = w_t/Z_t$. The price Phillips curve becomes: $\Pi_t^P(\Pi_t^P - 1) = \frac{\varepsilon-1}{\kappa}(\frac{w_t}{Z_t} - 1) + \beta\mathbb{E}_t\left[\Pi_{t+1}^P(\Pi_{t+1}^P - 1)\frac{Y_{t+1}}{Y_t}\right]$. The individual budget constraints on the two islands can be expressed as follows ($i = 1, 2$): $a_i + c_{i,t} = \frac{R_{t-1}^N}{\Pi_t^P}\tilde{a}_i + wy_i + \frac{y_i^\nu}{\Psi}\Omega_t$, with $\Psi = \frac{1}{2}(y_1^\nu + y_2^\nu)$. As the family head in each island cares equally about their island members, the two Euler equations can be written as:

$$u'(c_{1,t}) > \beta\mathbb{E}_t\frac{R_t^N}{\Pi_{t+1}^P}\left(\pi_{11}u'(c_{1,t+1}) + \pi_{12}u'(c_{2,t+1})\right), \tag{125}$$
$$u'(c_{2,t}) = \beta\mathbb{E}_t\frac{R_t^N}{\Pi_{t+1}^P}\left(\pi_{21}u'(c_{1,t+1}) + \pi_{22}u'(c_{2,t+1})\right).$$

[27] This flexible form allows one to encompass the case where profits are equally distributed to households (Acharya et al., 2023) and the case of an exogenous (unequal) distribution (Bhandari et al., 2021) without introducing another asset.

[28] The simple island metaphor is used by Lucas (1990), the family metaphor is also used in Challe et al. (2017) and Bilbiie and Ragot (2021) or Bilbiie (2024). See Ragot (2018) for an overview of limited-heterogeneity models.

The first Euler equation has to be a strict inequality for credit constraints to bind for type-1 agents. Using a utilitarian welfare function, the Ramsey program can be written as:

$$\max_{\left(c_{1,t},c_{2,t},w_t,R_t^N,\Pi_t^P\right)_{t\geq 0}} \frac{1}{2}\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t (u(c_{1,t}) + u(c_{2,t})),$$

$$\text{s.t.} \qquad c_{1,t} = \frac{R_{t-1}^N}{\Pi_t^P}\tilde{a}_1 - a_1 + w_t y_1 + \frac{y_1^\nu}{\Psi}\left(1 - \frac{w_t}{Z_t} - \frac{\kappa}{2}\left(\Pi_t^P - 1\right)^2\right) Z_t,$$

$$c_{2,t} = \frac{R_{t-1}^N}{\Pi_t^P}\tilde{a}_2 - a_2 + w_t y_2 + \frac{y_2^\nu}{\Psi}\left(1 - \frac{w_t}{Z_t} - \frac{\kappa}{2}\left(\Pi_t^P - 1\right)^2\right) Z_t,$$

$$u'(c_{2,t}) = \beta\mathbb{E}_t \frac{R_t^N}{\Pi_{t+1}^P}\left(\pi_{21}u'(c_{1,t+1}) + \pi_{22}u'(c_{2,t+1})\right),$$

$$\Pi_t^P(\Pi_t^P - 1)Z_t = \frac{\varepsilon - 1}{\kappa}(\frac{w_t}{Z_t} - 1)Z_t + \beta\mathbb{E}_t\Big[\Pi_{t+1}^P(\Pi_{t+1}^P - 1)Z_{t+1}\Big].$$

Denoting by $\lambda_{2,t}$ the Lagrange multiplier on the Euler equation of agents 2, and by γ_t the Lagrange multiplier on the Phillips curve, the Lagrangian associated to the Ramsey program is:

$$\mathcal{L} = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \Bigg[(u(c_{1,t}) + u(c_{2,t})) - (\gamma_t - \gamma_{t-1})\Pi_t^P(\Pi_t^P - 1)Z_t + \frac{\varepsilon - 1}{\kappa}\gamma_t(\frac{w_t}{Z_t} - 1)Z_t, \tag{126}$$

$$- \left(\lambda_{2,t} - \frac{R_{t-1}^N}{\Pi_t^P}\pi_{22}\lambda_{2,t-1}\right) u'(c_{2,t}) - \left(0 - \frac{R_{t-1}^N}{\Pi_t^P}\pi_{21}\lambda_{2,t-1}\right) u'(c_{1,t})\Bigg],$$

with $\lambda_{2,-1} = 0$. We denote by $\tilde{\lambda}_{1,t} = \pi_{21}\lambda_{2,t-1}$ and $\tilde{\lambda}_{2,t} = \pi_{22}\lambda_{2,t-1}$ the beginning-of-period Lagrange multipliers in each state.

The FOC with respect to Π_t^P is:

$$0 = \left(\psi_{1,t}\frac{y_1^\nu}{\Psi} + \psi_{2,t}\frac{y_2^\nu}{\Psi}\right)\kappa(\Pi_t^P - 1)Z_t + \frac{R_{t-1}^N}{(\Pi_t^P)^2}\left(\psi_{1,t}\tilde{a}_{1,t} + \psi_{2,t}\tilde{a}_{2,t} + \tilde{\lambda}_{1,t}u'(c_{1,t}) + \tilde{\lambda}_{2,t}u'(c_{2,t})\right)$$
$$+ (\gamma_t - \gamma_{t-1})(2\Pi_t^P - 1)Z_t. \tag{127}$$

The FOC with respect to R_t^N is:

$$0 = \beta\mathbb{E}_t \frac{1}{\Pi_{t+1}^P}\left(\psi_{1,t+1}\tilde{a}_1 + \psi_{2,t+1}\tilde{a}_2 + \tilde{\lambda}_{1,t+1}u'(c_{1,t+1}) + \tilde{\lambda}_{2,t+1}u'(c_{2,t+1})\right). \tag{128}$$

The FOC with respect to w_t is: $0 = \left(y_1 - \frac{y_1^\nu}{\Psi}\right)\psi_{1,t} + \left(y_2 - \frac{y_2^\nu}{\Psi}\right)\psi_{2,t} + \frac{\varepsilon-1}{\kappa}\gamma_t$.

We deduce from equations (127) and (128) that the steady-state allocation implies $\Pi^P = 1$. We can then compute the steady-state allocation and then deduce the steady-state values of Lagrange multipliers from previous equations. We also check that type-1 agents are indeed credit-constrained in equilibrium, i.e., that inequality (125) holds. The model after a TFP shock can then be simulated using a first-order perturbation of the model equations.

E.2.2 Quantitative assessment in the model without capital

The model calibration is close to the one of Bhandari et al. (2021). The period is a year (unlike in the rest of the paper). The preference parameters are set to $\beta = 0.96$ for the discount factor and to $\sigma = 2$ for the inverse of the intertemporal elasticity of substitution. The parameters characterizing the Phillips curve are set to $\varepsilon = 6$ (to have a markup of 20%) and to $\kappa = 20$, which generates a slope of the Phillips curve equal to 6%. The productivity process is based on an AR(1) process with a persistence of $\rho_\theta = 0.992$ and a standard deviation of the innovation of $\sigma_\theta = 10.3\%$. Using the Rouwenhorst (1995) procedure, we obtain the two productivity levels $y_1 = 0.11$ and $y_2 = 0.89$ and a transition matrix defined by $\pi_{11} = \pi_{22} = 0.996$ and $\pi_{12} = \pi_{21} = 1 - \pi_{11}$. We set the parameter driving the inequality of profits distribution to $\nu = 10$. This high value implies that the high-productivity agents get almost all the profits.[29] Finally, we set the credit constraint to $\bar{a} = 0.05$, which is half of the yearly income of low-productivity agents. This value implies that low-productivity agents are credit constrained in equilibrium. We solve for the optimal monetary policy after a negative TFP shock of 1%,with a persistence of $\rho_z = 0.73$, as in Bhandari et al. (2021).

As summary statistics, we report in Panel A of Figure 6 the inflation rate in percent and the consumption of agents 1 and 2, both in percentage deviation from their steady-state value. To understand the optimal allocation, we report the optimal response in black solid lines, and in blue dashed lines, the response with a constant inflation rate $\Pi^P = 1$, which corresponds to the optimal inflation rate in a representative-agent economy. The inflation increases at impact by around 0.17%, which is consistent with Bhandari et al. (2021). The increase in inflation benefits to low-productivity agents (agents 1) due to the Fisher effect and to the increase in the real wage. Agents 2 experience a slightly higher fall in consumption due to inflation. Overall, inflation acts as risk-sharing device for the aggregate TFP shock.

Parameter value	Description	Inflation on impact (in%)
Benchmark		0.17
$\kappa = 100$	Price adj. cost	0.11
$\sigma = 1$	IES	0.07
$\bar{a} = 0.1$	Credit limit	0.14
$\nu = 0$	lump-sum profit red.	-0.13

Table 10: Change in inflation on impact for different parameter values.

We conduct sensitivity analysis and report in Table 10 the increase in inflation at the time of impact for different parameter choices (the shape of the inflation response being mostly unchanged). The variation of the first inflation driver cost (κ increasing from 20 to 100) implies a

[29] The share of profits held by the type-1 agents is around $8 \cdot 10^{-10}$. A similar assumption is made in a number of papers (Challe et al., 2017, or Tobias et al., 2020, and see the discussion in Bilbiie, 2024).

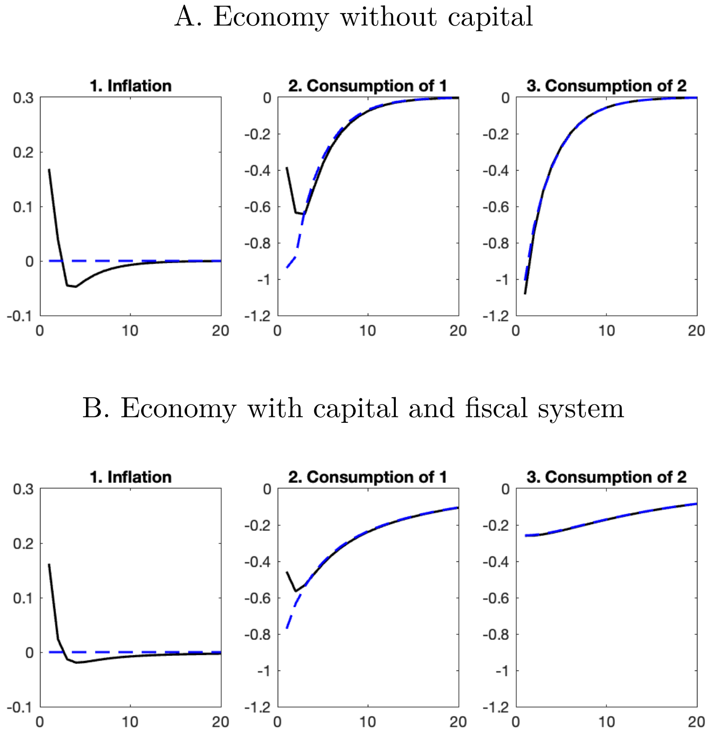

Figure 6: Optimal inflation rate in percentage point and consumption levels of the two types of agents in percentage deviation, for two economies. The black solid line is the optimal allocation. The blue dashed line is the allocation when $\Pi^P = 1$, which is the optimal inflation in the corresponding representative-agent economy.

decrease in the slope of the Phillips curve from 6% to the value of 2%. The increase in inflation on impact diminishes from 0.17% to 0.1%, consistent with a higher cost for inflation. With a higher IES (equal to $1/\sigma$), the welfare benefit of consumption smoothing is reduced, which implies a lower increase in inflation, by 0.07% instead of 0.17%. A higher credit limit increases the nominal outstanding debt of agents 1, which serve as a "fiscal" base of inflation. Finally, when the profits are equally distributed across agents (which corresponds to $\nu = 0$), inflation decreases on impact by -0.13% instead of increasing by 0.17%. Indeed, decreasing inflation allows the planner to increase firms' profits. Since the profits are equally distributed, this acts as a progressive transfer to type-1 agents (who are credit-constrained), which boosts their consumption. The sensitivity in a heterogeneous-agent economy of optimal inflation to profits distribution has already been documented (Tobias et al., 2020; Bhandari et al., 2021; Bilbiie and Ragot, 2021; Acharya et al., 2023; or Bilbiie, 2024).

E.2.3 Capital and taxes in the simple model

We consider the program with capital and exogenous taxes (possibly null). The production function is now $Y_t = Z_t K_t^\alpha$, as labor is still normalized to $L_t = 1$. Again, we guess-and-verify that agents 2 hold the capital stock, while agents 1 are credit-constrained. The pooling mechanism on islands now concerns both nominal and real assets.

The government finances a lump-sum transfer T_t by raising a distorting capital tax, τ_t^K, which can be time-varying. The post-tax gross real return on nominal bonds is $1 + (1-\tau_t^K)(\frac{\tilde{R}_{t-1}^N}{\Pi_t^P} - 1)$, while the post-tax gross real return on capital is $1 + (1-\tau_t^K)\tilde{r}_t^K$, where, as in Section 2, $\tilde{r}_t^K$ stands for the before-tax real interest rate on capital. The governmental budget constraint is:

$$T_t = \tau_t^K \left(\tilde{r}_t^K K_{t-1} + \left(\frac{\tilde{R}_{t-1}^N}{\Pi_t^P} - 1 \right) (\tilde{a}_1 + \tilde{a}_2) \right), \tag{129}$$

where $\tilde{a}_1 + \tilde{a}_2 = 0$, because nominal assets are in zero net-supply. The two budget constraints and the nominal and real Euler equations can be expressed as follows:

$$\begin{aligned}
-\bar{a} + c_{1,t} &= (1 + (1-\tau_t^K)(\frac{\tilde{R}_{t-1}^N}{\Pi_t^P} - 1))\tilde{a}_1 + (1 + (1-\tau_t^K)\tilde{r}_t^K)\tilde{K}_{1,t} + w_t y_1 + \frac{y_1^\nu}{\Psi}\Omega_t + T_t, \\
\bar{a} + K_t + c_{2,t} &= (1 + (1-\tau_t^K)(\frac{\tilde{R}_{t-1}^N}{\Pi_t^P} - 1))\tilde{a}_2 + (1 + (1-\tau_t^K)\tilde{r}_t^K)\tilde{K}_{2,t} + w_t y_2 + \frac{y_2^\nu}{\Psi}\Omega + T_t, \\
u'(c_{2,t}) &= \beta \mathbb{E}_t (1 + (1-\tau_{t+1}^K)(\frac{\tilde{R}_t^N}{\Pi_{t+1}^P} - 1)) \left(\pi_{21} u'(c_{1,t+1}) + \pi_{22} u'(c_{2,t+1}) \right), \\
u'(c_{2,t}) &= \beta \mathbb{E}_t (1 + (1-\tau_{t+1}^K)\tilde{r}_{t+1}^K) \left(\pi_{21} u'(c_{1,t+1}) + \pi_{22} u'(c_{2,t+1}) \right),
\end{aligned}$$

with the pooling expressions: $\tilde{K}_{1,t} = \pi_{21} K_{t-1}$ and $\tilde{K}_{2,t} = \pi_{22} K_{t-1}$. The Phillips curve is still given by (7). We denote by $\lambda_{k,2,t}$ the Lagrange multiplier on the Euler equation for capital, and by $\lambda_{b,2,t}$ the Lagrange multiplier on the Euler equation for the nominal asset. The beginning-of-period values of Lagrange multipliers on capital are $\tilde{\lambda}_{k,i,t} = \pi_{2,i}\lambda_{k,2,t}$, for $i = 1, 2$.

The Lagrange multiplier on the definition of ζ is Υ_t, and the one on the governmental budget constraint (129) is μ_t. The FOCs associated to the Ramsey program can now be easily derived.

Derivative with respect to $\tilde{R}_t^N$.

$$0 = \mathbb{E}_t \frac{1 - \tau_{t+1}^K}{\Pi_{t+1}^P} \left[\left(\psi_{t+1}^1 \tilde{a}_1 + \psi_{t+1}^2 \tilde{a}_2 \right) + \tilde{\lambda}_{b,2,t+1} u'(c_{2,t+1}) + \tilde{\lambda}_{b,1,t+1} u'(c_{1,t+1}) \right].$$

Derivative with respect to $\tilde{r}_t^K$.

$$\begin{aligned}
0 =& (\psi_{1,t}\tilde{K}_{1,t} + \psi_{2,t}\tilde{K}_{2,t} + \tilde{\lambda}_{1,t}^K u'(c_{1,t}) + \tilde{\lambda}_{2,t}^K u'(c_{2,t}))(1-\tau_t^K)(\tilde{r}_t^K + \delta) \\
&+ \left(\frac{\varepsilon - 1}{\kappa}\gamma_t - \mu_t^P \right) \alpha \zeta_t Z_t K_{t-1}^\alpha - (1-\alpha)\Upsilon_t \zeta_t + (\tilde{r}_t^K + \delta)\mu_t \tau^K K_{t-1}.
\end{aligned}$$

Derivative with respect to w_t.

$$0 = (y_1\psi_{1,t} + y_1\psi_{2,t})w_t + \left(\frac{\varepsilon-1}{\kappa}\gamma_t - (\frac{y_1^\nu}{\Psi}\psi_{1,t} + \frac{y_2^\nu}{\Psi}\psi_{2,t})\right)(1-\alpha)\zeta_t Z_t K_{t-1}^\alpha + (1-\alpha)\zeta_t \Upsilon_t.$$

Derivative with respect to Π_t^P.

$$\begin{aligned}0 = {} & (1-\tau_t^K)\frac{\tilde{R}_{t-1}^N}{(\Pi_t^P)^2}\left(\psi_{1,t}\tilde{a}^1 + \psi_{2,t}\tilde{a}^2 + \tilde{\lambda}_{b,1,t}u'(c_{1,t}) + \tilde{\lambda}_{b,2,t}u'(c_{2,t})\right)\\ & + (\gamma_t - \gamma_{t-1})(2\Pi_t^P - 1)Z_t K_{t-1}^\alpha + \left(\frac{y_1^\nu}{\Psi}\psi_{1,t} + \frac{y_2^\nu}{\Psi}\psi_{2,t}\right)\kappa(\Pi_t^P - 1)Z_t K_{t-1}^\alpha.\end{aligned}$$

Derivative with respect to T_t.

$$\mu_t = \psi_{1,t} + \psi_{2,t}.$$

Derivative with respect to K_t.

$$\begin{aligned}\psi_{2,t} = {} & \beta\mathbb{E}_t(1 + (1-\tau_{t+1}^K)\tilde{r}_{t+1}^K)(\pi_{22}\psi_{2,t+1} + \pi_{21}\psi_{1,t+1})\\ & - \alpha\beta\mathbb{E}_t\left[(\gamma_{t+1} - \gamma_t)\Pi_{t+1}^P(\Pi_{t+1}^P - 1) - \frac{\varepsilon-1}{\kappa}\gamma_{t+1}(\zeta_{t+1} - 1)\right]Z_{t+1}K_t^{\alpha-1}\\ & + \alpha\beta\mathbb{E}_t\left(\frac{y_1^\nu}{\Psi}\psi_{1,t+1} + \frac{y_2^\nu}{\Psi}\psi_{2,t+1}\right)(1 - \zeta_{t+1} - \frac{\kappa}{2}(\Pi_{t+1}^P - 1)^2)Z_{t+1}K_t^{\alpha-1}\\ & - \beta\mathbb{E}_t\Upsilon_{t+1}\frac{1-\alpha}{\alpha Z_{t+1}}(\tilde{r}_{t+1}^K + \delta)K_t^{-\alpha} + \beta\mathbb{E}_t\mu_{t+1}\tau_{t+1}^K\tilde{r}_{t+1}^K.\end{aligned}$$

The steady state corresponds to $\Pi^P = 1$, $T = \tau^K = 0$, and $1 + \tilde{r}^K = \tilde{R}^N$. We can then compute the steady-state allocation, as well as the steady-state values of Lagrange multipliers. The model dynamics can then be simulated using perturbation methods.

E.2.4 Quantitative assessment of the model with capital and taxes

We extend the calibration of Section E.2.2 (still with an annual period). For the production, we set the capital share to $\alpha = 1/3$ and the annual depreciation rate to $\delta = 0.1$. Production factors are determined as in Section 2.3 together with $L_t = 1$. We assume that the capital tax is countercyclical and follows the simple rule $\tau_t^K = -\sigma^k z_t$, with $\sigma^k = 0.5$. This rule implies that the capital tax, as well as the lump-sum transfers, are null at the steady state, positive in recession, but negative in booms. The outcome is reported in Panel B of Figure 6. Inflation barely moves on impact and in the dynamics. Time-varying redistribution across agents is now done by the tax system, and the inflation now has a modest role in the distribution of resources.

We can draw two main conclusions from our simple model exercise. The first is that monetary policy can play an active role as a risk-sharing tool (see Panel A of Figure 6). The optimal inflation response is nevertheless sensitive to the calibration, and especially to the inequality of profit distribution, the slope of the Phillips curve, and the IES. Second, introducing a simple taxation scheme that allows for redistribution basically turns off monetary policy. The inflation response becomes much smaller in the presence of fiscal tools (see Panel B of Figure 6). In other words, inflation allows for redistribution, but is a very costly substitute for fiscal policy.

E.3 Unequal profit distribution

We verify that the conclusions of Section E.2, remain valid in a quantitative model, similar to the one of Sections 4 and 5, with the notable exception that profits are directly distributed to agents and not taxed away by the government. As in Section E.2, agents of type i receive a profit amount equal to $(\sum S_y y)^{-1} y_i^{\nu} \Omega_t$, where $\nu \geq 0$ drives how unequal the profit distribution is. Compared to Section 4, this modifies the budget constraints of the government and of households, and hence the derivation of the Ramsey program.

E.3.1 The Ramsey program

The Ramsey program is similar to the one of Section 4.3, except that the budget constraints of households and of the government (equations (45) and (48)) are modified as follows:

$$a_{i,t} + c_{i,t} = (1 + r_t)a_{i,t-1} + w_t y_{i,t} l_{i,t} + T_t + \frac{y_{i,t}^{\nu}}{\sum_y S_y y^{\nu}} \left(1 - \zeta_t - \frac{\kappa}{2}(\Pi_t^P - 1)^2\right) Y_t,$$

$$B_t + \zeta_t Y_t - \delta K_{t-1} = G_t + B_{t-1} + r_t A_{t-1} + w_t L_t + T_t.$$

For space reasons, we directly present the FOCs of the truncated model. The SVL $\psi_{t,h}$ is still defined in (103). These FOCs with respect to $\tilde{R}_t^N$, r_t, and T_t are unchanged compared to the full profit taxation of Section 4.3 and correspond to equations (83), (104), and (106), respectively. The other FOCs are modified with new profit distribution.

FOC with respect to Π_t^P.

$$0 = \frac{\kappa(\Pi_t^P - 1)}{\sum_y S_y y^{\nu}} \sum_{h \in \mathcal{H}} S_{t,h} (y_0^h)^{\nu} \psi_{t,h} + (\gamma_t - \gamma_{t-1})(2\Pi_t^P - 1) + \left(\beta^{-1} \Upsilon_{t-1} - \Gamma_t (1 - \tau_t^K) B_{t-1}\right) \frac{\tilde{R}_{t-1}^N}{Y_t \left(\Pi_t^P\right)^2}. \tag{130}$$

FOC with respect to ζ_t.

$$0 = w_t \sum_{h \in \mathcal{H}} S_{t,h} y_0^h \left(l_{t,h} \psi_{t,h} + \lambda_{l,t,h} u'(c_{t,h})\right) - (\tilde{r}_t^K + \delta) \left(\beta^{-1} \Upsilon_{t-1} + \Gamma_t (1 - \tau_t^K) K_{t-1}\right) + \left(\frac{\varepsilon - 1}{\kappa} \gamma_t + \mu_t - \frac{1}{\sum_y S_y y^{\nu}} \sum_{h \in \mathcal{H}} S_{t,h} (y_0^h)^{\nu} \psi_{t,h}\right) \zeta_t Y_t - w_t \mu_t L_t. \tag{131}$$

FOC with respect to $a_{t,h}$. For unconstrained agents:

$$\begin{aligned}
\psi_{t,h} =& \beta\mathbb{E}_t\left[(1+r_{t+1})\sum_{\tilde{h}\in\mathcal{H}}\pi_{h\tilde{h}}\hat{\psi}_{t+1,\tilde{h}}\right] + \beta\mathbb{E}_t\left[\Gamma_{t+1}(r_{t+1}-(1-\tau^K_{t+1})\tilde{r}^K_{t+1})\right] \qquad (132)\\
&+\beta\frac{\alpha}{K_t}\mathbb{E}_t\left[w_{t+1}\sum_{h\in\mathcal{H}}S_{t+1,h}(y^h_0)^\nu\left(\psi_{t+1,h}l_{t+1,h}+\lambda_{l,t+1,h}u'(c_{t+1,h})\right)\right]\\
&+\beta\frac{\alpha}{K_t}\mathbb{E}_t\left[\mu_{t+1}\left(Y_{t+1}\zeta_{t+1}-\frac{1}{\alpha}(r_{t+1}+\delta)K_t-w_{t+1}L_{t+1}\right)\right]\\
&-\beta\frac{\alpha}{K_t}\mathbb{E}_t\left[\left((\gamma_{t+1}-\gamma_t)\Pi^P_{t+1}(\Pi^P_{t+1}-1)-\frac{\varepsilon-1}{\kappa}\gamma_{t+1}(\zeta_{t+1}-1)\right)Y_{t+1}\right]\\
&-\beta\frac{\alpha-1}{K_t}\mathbb{E}_t\left[(\tilde{r}^K_{t+1}+\delta)\left(\beta^{-1}\Upsilon_t+\Gamma_{t+1}(1-\tau^K_{t+1})K_t\right)\right]\\
&+\beta\frac{\alpha}{K_t}\mathbb{E}_t\left[Y_{t+1}\frac{1}{\sum_y S_y y^\nu}\left(1-\zeta_{t+1}-\frac{\kappa}{2}(\Pi^P_{t+1}-1)^2\right)\sum_{h\in\mathcal{H}}S_{t+1,h}(y^h_0)^\nu\psi_{t+1,h}\right],
\end{aligned}$$

while for constrained agents, we have $a_{t,h} = -\bar{a}$ and $\lambda_{t,h} = 0$.

FOC with respect to $l_{t,h}$.

$$\begin{aligned}
\psi_{t,h} =& \frac{1}{w_t y^h_0}v'(l_{t,h}) + \frac{\lambda_{l,t,h}}{w_t y^h_0}v''(l_{t,h}) - (1-\alpha)\frac{\mu_t}{w_t L_t}\left(Y_t\zeta_t - w_t L_t\right)\\
&+(1-\alpha)\frac{Y_t}{w_t L_t}\left((\gamma_t-\gamma_{t-1})\Pi^P_t(\Pi^P_t-1)-\frac{\varepsilon-1}{\kappa}\gamma_t(\zeta_t-1)\right)\\
&+(1-\alpha)\frac{\tilde{r}^K_t+\delta}{w_t L_t}\left(\beta^{-1}\Upsilon_{t-1}+\Gamma_t(1-\tau^K_t)K_{t-1}\right)\\
&+\alpha\frac{1}{L_t}\sum_{h\in\mathcal{H}}S_{t,h}y^h_0\left(l_{t,h}\psi_{t,h}+\lambda_{l,t,h}u'(c_{t,h})\right)\\
&-(1-\alpha)\frac{Y_t}{w_t L_t}\frac{1}{\sum_y S_y y^\nu}\left(1-\zeta_t-\frac{\kappa}{2}(\Pi^P_t-1)^2\right)\sum_{h\in\mathcal{H}}S_{t,h}(y^h_0)^\nu\psi_{t,h}.
\end{aligned}$$

The matrix representation to solve for the steady-state values of Lagrange multiplier, is similar to the one Section D.5, with one exception, which is the steady-state value of the Lagrange multiplier, γ, and which becomes:

$$\begin{aligned}
\frac{\varepsilon-1}{\kappa}\gamma Y =& Y\left(\frac{(\boldsymbol{y}^\nu)^\top(\boldsymbol{S}\circ\boldsymbol{\psi})}{(\boldsymbol{y}^\nu)^\top\boldsymbol{S}}-\mu\right)+(\tilde{r}^K+\delta)\left(\beta^{-1}\Upsilon+\Gamma_t(1-\tau^K)K\right)+w\mu L\\
&-w(\boldsymbol{y}\circ\boldsymbol{l})^\top(\boldsymbol{S}\circ\boldsymbol{\psi})-w(\boldsymbol{y}\circ u'(\boldsymbol{c}))^\top(\boldsymbol{S}\circ\boldsymbol{\lambda}_l).
\end{aligned}$$

E.3.2 The quantitative exercise

We now show that a modification of our baseline model, targeting same moments as Bhandari et al. (2021) generate consistent results compared to theirs. We keep the same quarterly calibration as in Section 5, except that the Rotemberg cost parameter is set to $\kappa = 20$, instead of 100 in the baseline quarterly calibration. This implies a slope of the quarterly Phillips curve equal to 6%, as in Bhandari et al. (2021). A value $\kappa = 20$ implies an average price duration in the Calvo model of 2.5 quarters (whereas a value of $\kappa = 100$ implies an average price duration of 5 quarters).

Furthermore, the parameter ν driving how unequal the profit distribution is, is set to $\nu = 2.3$ to match the inflation movement in Bhandari et al. (2021). This implies that the most productive agents hold 55% of profits.

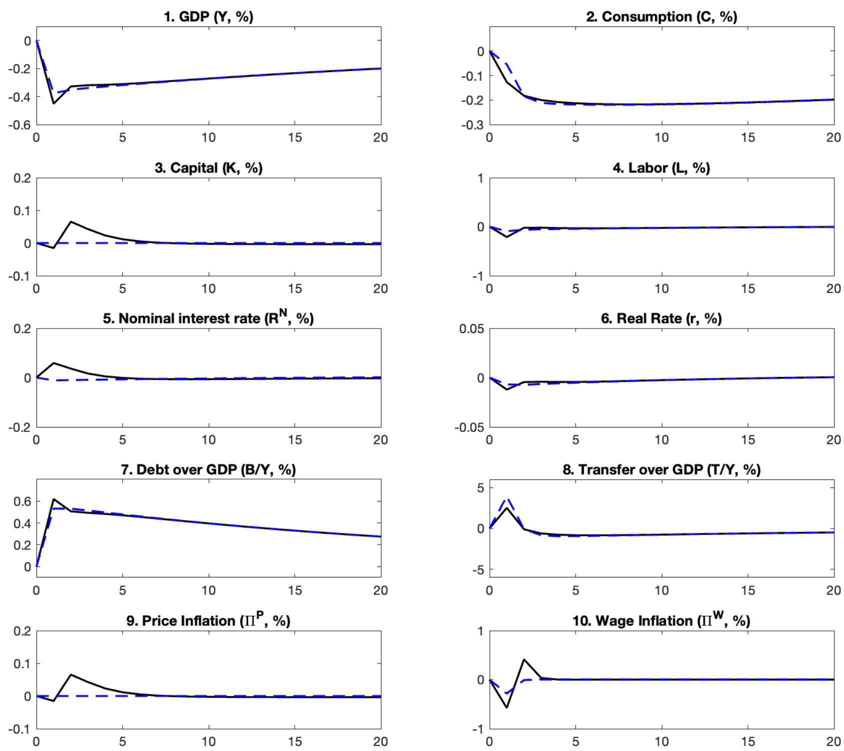

Figure 7: IRFs after a negative productivity shock for relevant variables. The black line is Economy $1'$ with acyclical fiscal policy and optimal monetary policy. The blue dashed line is Economy $2'$ with acyclical fiscal policy and $\Pi_t^P = 1$.

We consider three economies that we denote with a prime compared to their counterpart from the main text. Economies $1'$ and $2'$ are the parallel of Economy 1 and 2 of Section 5, but with the updated calibration. They both implement the acyclical fiscal rule, but with either optimal monetary policy (Economy $1'$) or a constant inflation $\Pi_t^P = 1$ (Economy $2'$). We report the IRFs after a negative productivity shock of one standrad deviation in Figure 7. These IRFs confirm the finding of the simple models of Section E.2: a large slope of the Phillips curve, and an unequal profit distribution are key to generate a sizable inflation response. The maximal quarterly inflation response approximately amounts to 0.065%, or 0.26% annually – which is the same value as in Bhandari et al. (2021).

F Comparisons with the Reiter method

Since Section D.7 has shown that the uniform and refined truncation methods yield similar results, we compare the Reiter method to the refined truncation method of Section D.7 (with $N = 5$ and a refinement length of 20). We assume full price flexibility and no public spending ($G = 0$), and hence no tax. The rest of the calibration ("preference and technology" and "shock process") is the same as in the baseline calibration of Table 1. We still use 5 idiosyncratic states.

First, Figure 8 plots the IRFs for the main variables after a negative TFP shock of one standard deviation. The two models are very close, and almost indistinguishable. Second, we

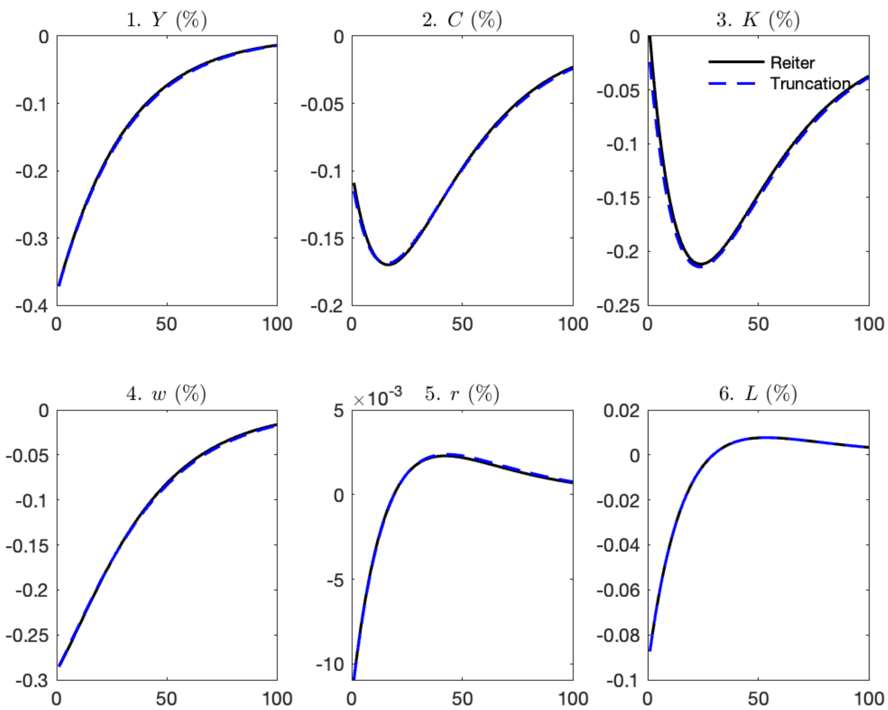

Figure 8: IRFs after a negative TFP shock of one standard deviation, in percentage deviation for all variables, except r in level deviation. The black solid line is Reiter method and red dashed line is truncation method.

report first and second-order moments. First-order moments are similar by construction, as the two methods involve a perturbation method around the same steady state. Normalized standard deviations are reported in percent to ease the reading. Again, second-order moments are very close. The difference is around 10^{-4}, which is a good outcome.

		Reiter	Truncation
Y	Mean	1.4773	1.4773
	Std(%)	1.4369	1.4524
C	Mean	1.0252	1.0252
	Std(%)	1.0558	1.0598
K	Mean	18.0831	18.0832
	Std(%)	1.3338	1.3667
L	Mean	0.3610	0.3610
	Std(%)	0.2110	0.2216
r	Mean	0.0044	0.0044
	Std(%)	0.0280	0.0286
w	Mean	2.6187	2.6187
	Std(%)	1.2927	1.2938
Correlations			
$corr(C, Y)$		0.9064	0.9130
$corr(Y, Y_{-1})$		0.9668	0.9672

Table 11: First- and second-order moments for key variables, comparing the same model simulated with the Reiter's method and the refined truncation method.

Third, we simulate the model for 10,000 periods to report the average and maximum relative differences (i.e. divided by the mean of the variables) for Y, C, K, L, w, and absolute difference for r between the two simulation methods. Again these results confirm that the two simulation methods generate very close results.

	Y	C	K	L	r	w
Average	$1.60 \cdot 10^{-4}$	$1.37 \cdot 10^{-4}$	$4.73 \cdot 10^{-4}$	$1.36 \cdot 10^{-4}$	$6.37 \cdot 10^{-6}$	$1.22 \cdot 10^{-4}$
Maximum	$8.34 \cdot 10^{-4}$	$6.54 \cdot 10^{-4}$	$2.45 \cdot 10^{-3}$	$6.63 \cdot 10^{-5}$	$3.51 \cdot 10^{-5}$	$6.72 \cdot 10^{-4}$

Table 12: Average and maximum absolute differences for key variables between the two methods for a simulation over 10,000 periods.

G Social weights and optimal fiscal policy

In this section, we explain how using social weights in the social welfare function enables us to jointly consider optimal fiscal and monetary policies – with a missing instrument to avoid the equivalence result of Section 3. We assume here that the capital tax does not vary along the business cycle and is fixed at its optimal steady-state value τ_{SS}^{K}.

We consider a social welfare function that departs from the utilitarian welfare. The utility functions of agents are heterogeneously weighted. We assume that these weights are consistent with the sequential representation, and depends on initial conditions and idiosyncratic and aggregate history. The weight of an agent $i \in \mathcal{I}$ is $\omega_{i,t} = \omega_t((a_{-1}, y_0), y_i^t, z^t)$, where the sequence of weights satisfy $1 = \int_i \omega_{i,t}\ell(di) = \sum_{y_i^t \in \mathcal{Y}^t} \sum_{y_0 \in \mathcal{Y}} \int_{a_{-1} \in [-\bar{a};+\infty)} \omega_t((a_{-1}, y_0), y_i^t, z^t)\theta_t(y_i^t)\Lambda(da_{-1}, y_0)$ for $t \geq 0$. These weights represent the relative importance of each agent in the planner's objective and will be calibrated in our quantitative exercise below, so as to match the US fiscal and monetary policies at the steady-state. Formally, the aggregate welfare criterion is:

$$\widetilde{W}_0 = \mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta^t \int_i \omega_{i,t} \left(u(c_{i,t}) - v(l_{i,t}) \right) \ell(di) \right]. \tag{133}$$

We consider the same setup as in Section 4, except that taxes are not determined through rules (50)–(52), and that the planner's objective is defined in (133). The Ramsey planner's program now involves the labor tax and is: $\max_{\left(\tau_t^L, T_t, w_t, r_t, \tilde{w}_t, \tilde{r}_t^K, \tilde{R}_t^N, B_t, K_t, L_t, \Pi_t^P, (a_{i,t}, c_{i,t}, l_{i,t}, \nu_{i,t})_i\right)_{t\geq 0}} \widetilde{W}_0$, subject to: $\tau_t^k = \tau_{SS}^k$, and the same constraints as in Section 4.3 (but the fiscal rules).

To save some space, we directly solve for the truncated model.

G.1 The truncated Ramsey model

The truncated Ramsey program can be written as:

$$\max_{\left((a_{t,h}, c_{t,h}, l_{t,h})_{h\in\mathcal{H}}, w_t, r_t, \tilde{w}_t, \tilde{r}_t, B_t, T_t, \Pi_t^P\right)_{t\geq 0}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \sum_{h \in \mathcal{Y}} \left[S_{t,h} \left(\omega_h \left(\xi_h^{u,0} u(c_{t,h}) - \xi_h^{v,0} v(l_{t,h}) \right) \right.\right.$$
$$\left.\left. -(\lambda_{t,h} - \tilde{\lambda}_{t,h}(1+r_t))\xi_h u'(c_{t,h}) - \lambda_{l,t,h} \left(\xi_{t,h}^{v,1} v'(l_{t,h}) - w_t y_{t,h} \xi_h^{u,1} u'(c_{t,h}) \right) \right) \right],$$

subject to truncated-history constraints:

$$c_{t,h} + a_{t,h} = w_t y_0^N l_{t,h} + (1+r_t)\tilde{a}_{t,h} + T_t, \text{ and } \tilde{a}_{t,h} = \sum_{\tilde{h} \in \mathcal{H}} \pi_{\tilde{h}h} \frac{S_{t-1,\tilde{h}}}{S_{t,h}} a_{t-1,\tilde{h}}.$$

to aggregate constraints:

$$G_t + r_t \left(B_{t-1} + K_{t-1} \right) + w_t L_t = B_t - B_{t-1} - T_t + \left(1 - \kappa \frac{\pi_{2,t}}{2} \right) K_{t-1}^{\alpha} L_t^{1-\alpha} - \delta K_{t-1},$$
$$\Pi_t^P(\Pi_t^P - 1) = \frac{\varepsilon - 1}{\kappa}(\zeta_t - 1) + \beta \mathbb{E}_t \Pi_{t+1}^P (\Pi_{t+1}^P - 1) \frac{Y_{t+1}}{Y_t},$$
$$\zeta_t = \frac{1}{Z_t} \left(\frac{\tilde{r}_t^K + \delta}{\alpha} \right)^{\alpha} \left(\frac{\tilde{w}_t}{1-\alpha} \right)^{1-\alpha}, \; A_t = K_t + B_t = \sum_{h \in \mathcal{Y}} S_{t,h} a_{t,h}, \; L_t = \sum_{h \in \mathcal{Y}} S_{t,h} y_0^N l_{t,h},$$

and to interest rate constraints:

$$\mathbb{E}_t \left[\frac{R_t^N}{\Pi_{t+1}^P} \right] = \mathbb{E}_t \left[1 + \tilde{r}_{t+1}^K \right], \; (r_t - (1 - \tau_{SS}^K)\tilde{r}_t^K) A_{t-1} = (1 - \tau_{SS}^K)(\frac{\tilde{R}_{t-1}^N}{\Pi_t^P} - 1 - \tilde{r}_t^K) B_{t-1}. \tag{134}$$

The Lagrangian associated to the previous Ramsey program is:

$$\mathcal{L} = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \Bigg[\sum_{h\in\mathcal{Y}} S_{t,h} \Big(\omega_h \left(\xi_h^{u,0} u(c_{t,h}) - \xi_h^{v,0} v(l_{t,h}) \right) \\
- \sum_{h\in\mathcal{Y}} S_{t,h} (\lambda_{t,h} - \tilde{\lambda}_{t,h}(1+r_t)) \xi_h u'(c_{t,h}) - \sum_{h\in\mathcal{Y}} \lambda_{l,t,h} S_{t,h} \left(\xi_{t,h}^{v,1} v'(l_{t,h}) - w_t y_{t,h} \xi_h^{u,1} u'(c_{t,h}) \right) \\
- (\gamma_t - \gamma_{t-1}) \Pi_t^P (\Pi_t^P - 1) Y_t + \frac{\varepsilon - 1}{\kappa} \gamma_t (\frac{1}{Z_t} \left(\frac{\tilde{r}_t^K + \delta}{\alpha} \right)^{\alpha} \left(\frac{\tilde{w}_t}{1-\alpha} \right)^{1-\alpha} - 1) Y_t \\
+ \mu_t \left(B_t - (1-\delta) B_{t-1} - G_t - T_t + (1 - \frac{\kappa}{2} (\Pi_t^P - 1)^2) Y_t - (r_t + \delta) A_{t-1} - w_t L_t \right) \\
+ \frac{1}{\beta} \Upsilon_{t-1} (\frac{\tilde{R}_{t-1}^N}{\Pi_t^P} - (1 + \tilde{r}_t^K)) + \Gamma_t (r_t A_{t-1} - (1 - \tau_t^K)(\tilde{r}_t^K K_{t-1} + (\frac{\tilde{R}_{t-1}^N}{\Pi_t^P} - 1) B_{t-1}) \Big) \\
+ \Lambda_t \left(\frac{\tilde{r}_t^K + \delta}{\alpha} K_{t-1} - \frac{\tilde{w}_t}{1-\alpha} L_t \right) \Bigg].$$

G.2 FOCs in the truncated model

We define the net SVL, $\hat{\psi}$, as follows:

$$\hat{\psi}_{t,h} = \omega_h \xi_h^{u,0} u'(c_{t,h}) - \left(\lambda_{c,t,h} \xi_h^{u,E} - (1+r_t) \tilde{\lambda}_{c,t,h} \xi_h^{u,E} - \lambda_{l,t,h} w_t y_0^N \xi_h^{u,1} \right) u''(c_{t,h}) - \mu_t.$$

Derivative with respect to $\tilde{R}_t^N$.

$$(1 - \tau_{SS}^K) \mathbb{E}_t \left[\frac{\Gamma_{t+1}}{\Pi_{t+1}^P} \right] B_t = \Upsilon_t \mathbb{E}_t \left[\frac{1}{\Pi_{t+1}^P} \right].$$

Derivative with respect to $\tilde{r}_t^K$.

$$\frac{1}{\beta} \Upsilon_{t-1} + \Gamma_t (1 - \tau_{SS}^K)(A_{t-1} - B_{t-1}) = \frac{\varepsilon - 1}{\kappa} \gamma_t K_{t-1} + \Lambda_t \frac{1}{\alpha} K_{t-1}.$$

Derivative with respect to Π_t^P.

$$0 = \mu_t \kappa (\Pi_t^P - 1) + (\gamma_t - \gamma_{t-1})(2\Pi_t^P - 1) - (\Gamma_t (1 - \tau_{SS}^K) B_{t-1} - \beta^{-1} \Upsilon_{t-1}) \frac{\tilde{R}_{t-1}^N}{Y_t \left(\Pi_t^P \right)^2}.$$

Derivative with respect to r_t.

$$\sum_{h\in\mathcal{H}} S_{t,h} \hat{\psi}_{t,h} \tilde{a}_{t,h} = - \sum_{h\in\mathcal{H}} S_{t,h} \tilde{\lambda}_{t,h} \xi_h U_c(c_{t,h}, l_{t,h}) - \Gamma_t A_{t-1}.$$

Derivative with respect to $\tilde{w}_t$.

$$\Lambda_t = (1 - \alpha) \frac{\varepsilon - 1}{\kappa} \gamma_t.$$

Derivative with respect to w_t.

$$0 = \sum_{h \in \mathcal{H}} S_{t,h} y_0^N (\hat{\psi}_{t,h} l_{t,h} + \lambda_{t,l,h} \xi_h^{u,1} u'(c_{t,h})).$$

Derivative with respect to B_t.

$$\begin{aligned}
\mu_t = {} & \beta \mathbb{E}_t \left[\mu_{t+1} \left(1 - \delta + \alpha \frac{Y_{t+1}}{K_t} \left(1 - \frac{\kappa}{2} (\Pi_{t+1}^P - 1)^2 \right) \right) \right] \\
& - \alpha \beta \mathbb{E}_t \left[((\gamma_{t+1} - \gamma_t) \Pi_{t+1}^P (\Pi_{t+1}^P - 1) + \frac{\varepsilon - 1}{\kappa} \gamma_{t+1}) \frac{Y_{t+1}}{K_t} \right] \\
& + \frac{\varepsilon - 1}{\kappa} \beta \mathbb{E}_t \left[\gamma_{t+1} \frac{\zeta_{t+1} Y_{t+1}}{K_t} \right] + \beta \mathbb{E}_t \Lambda_{t+1} \frac{\tilde{r}_{t+1}^K + \delta}{\alpha} + \beta (1 - \tau_{SS}^K) \mathbb{E}_t \left[\Gamma_{t+1} \left(\frac{\tilde{R}_t^N}{\Pi_{t+1}^P} - 1 - \tilde{r}_{t+1}^K \right) \right].
\end{aligned} \tag{135}$$

Derivative with respect to $a_{t,h}$. For unconstrained truncated history h:

$$\begin{aligned}
\psi_{t,h} = {} & \beta \mathbb{E}_t \left[(1 + r_{t+1}) \psi_{i,t+1} \right] + \beta \mathbb{E}_t \left[\mu_{t+1} \left(- (1 + r_{t+1}) + 1 - \delta + \alpha \frac{Y_{t+1}}{K_t} (1 - \frac{\kappa}{2} (\Pi_{t+1}^P - 1)^2) \right) \right] \\
& + \beta \mathbb{E}_t \left[\Gamma_{t+1} \left(r_{t+1} - (1 - \tau^K) \tilde{r}_{t+1}^K \right) \right] + \beta \mathbb{E}_t \left[\Lambda_{t+1} \frac{\tilde{r}_{t+1}^K + \delta}{\alpha} \right] + \beta \mathbb{E}_t \Lambda_{t+1} \left(\frac{\tilde{r}_{t+1}^K + \delta}{\alpha} \right) \\
& - \alpha \beta \mathbb{E}_t \left[\left((\gamma_{t+1} - \gamma_t) \Pi_{t+1}^P (\Pi_{t+1}^P - 1) - \frac{\varepsilon - 1}{\kappa} \gamma_{t+1} (\zeta_{t+1} - 1) \right) \frac{Y_{t+1}}{K_t} \right],
\end{aligned}$$

while for constrained ones: $a_{t,h} = -\bar{a}$ and $\lambda_{t,h} = 0$. Using (135) yields

$$\hat{\psi}_{t,h} = \beta \left[(1 + r_{t+1}) \sum_{h \in \mathcal{Y}} \pi_{h\tilde{h}} \hat{\psi}_{t+1,h} \right] + \beta \mathbb{E}_t \left[\Gamma_{t+1} (r_{t+1} - (1 - \tau_{t+1}^K) (\frac{\tilde{R}_t^N}{\Pi_{t+1}^P} - 1)) \right]. \tag{136}$$

FOC with respect to $l_{i,t}$.

$$\begin{aligned}
\hat{\psi}_{t,h} = {} & \frac{\omega_{t,h} \xi_{t,h}^v}{w_t y_h} v'(l_{t,h}) + \frac{\lambda_{l,t} \xi_{t,h}^v}{w_t y_h} v''(l_{t,h}) \\
& - (1 - \alpha) \frac{\mu_t}{w_t L_t} Y_t \left(1 - \frac{\kappa}{2} (\Pi_t^P - 1)^2 \right) - \Lambda_t \frac{\tilde{w}_t}{1 - \alpha} \frac{1}{w_t} \\
& + (1 - \alpha) \frac{1}{w_t L_t} \left((\gamma_t - \gamma_{t-1}) \Pi_t^P (\Pi_t^P - 1) - \frac{\varepsilon - 1}{\kappa} \gamma_t (\zeta_t - 1) \right) Y_t.
\end{aligned}$$

FOC with respect to T_t.

$$\sum_{h \in \mathcal{H}} S_{t,h} \hat{\psi}_{t,h} \leq 0, \tag{137}$$

with equality if $T_t > 0$.

G.3 Steady state

We drop the t subscript to denote steady-state variables. The optimality conditions imply $\Pi^P = \zeta = 1$ and $0 = \gamma = \Gamma = \Upsilon = \Lambda$, while equations characterizing the Ramsey allocation are:

$$\hat{\psi}_h = \omega_h \xi_h^{u,0} u'(c_h) - \left(\lambda_{c,h}\xi_h^{u,E} - (1+r_t)\tilde{\lambda}_{c,h}\xi_h^{u,E} - \lambda_{l,h} w_t y_0^N \xi_h^{u,1}\right) u''(c_h) - \mu, \quad (138)$$

$$\hat{\psi}_h = \beta(1+r)\sum \pi_{h\tilde{h}}\hat{\psi}_{\tilde{h}}, \text{ if } h \text{ unconstrained}, \lambda_h = 0, \text{ otherwise}, \quad (139)$$

$$\hat{\psi}_h = \frac{1}{wy_0^N}(\omega_h \xi_h^{v,0} v'(l_h) + \lambda_{l,h}\xi_h^{v,1} v''(l_h)) - \mu(1-\alpha)\frac{Y}{wL}, \quad (140)$$

$$0 = \sum_{h\in\mathcal{Y}} S_h \left(\hat{\psi}_h \tilde{a}_h + \tilde{\lambda}_{c,h}\xi_h^{u,E} u'(c_h)\right) = \sum_{h\in\mathcal{Y}} S_h l_h y_h \left(\hat{\psi}_h + \lambda_{l,h}\xi_h^{u,1}(u'(c_h)/l_h)\right) = \sum_{h\in\mathcal{Y}} S_h \hat{\psi}_h, \quad (141)$$

$$1 = \beta(1+F_K). \quad (142)$$

G.4 A closed-form formula for the weights ωs

We use the same matrix notation as in Section D.5.

G.4.1 Notation

We start with the various ξs. Using truncation:

$$\xi_h^{u,0} = \frac{\sum_h S_h u(c_h)}{u(c_h)},\ \xi_h^{u,1} = \frac{\sum_h S_h u'(c_h)}{u'(c_h)},\ \xi_h^{v,0} = \frac{\sum_h S_h v(c_h)}{v(c_h)},$$

while as in Section D.5, we use Euler equation to determine $\boldsymbol{\xi}^{u,E}$ and $\boldsymbol{\xi}^{v,1}$ as:

$$\boldsymbol{\xi}^{u,E} = \left[\left(\boldsymbol{I} - \beta(1+r)\boldsymbol{\Pi}^\top\right)\boldsymbol{D}_{u'(\boldsymbol{c})}\right]^{-1}\boldsymbol{\nu}, \quad (143)$$

$$\boldsymbol{\xi}^{v,1} = w\boldsymbol{y} \circ \boldsymbol{l} \circ \boldsymbol{\xi}^{u,1} \circ u'(\boldsymbol{c})./v'(\boldsymbol{l}), \quad (144)$$

where ./ denotes the element-wise division for vectors. We also introduce the following notation:

$$\tilde{\boldsymbol{\xi}}^{v,1} := \boldsymbol{\xi}^{v,1}./(w\boldsymbol{y}),\ \tilde{\boldsymbol{\xi}}^{v,0} := \boldsymbol{\xi}^{v,0}./(w\boldsymbol{y}),\ \tilde{\boldsymbol{\xi}}^{u,1} := \boldsymbol{\xi}^{u,1}./\boldsymbol{l},$$

as well as: $\bar{\lambda}_l := \boldsymbol{S} \circ \lambda_l,\ \bar{\lambda}_c := \boldsymbol{S} \circ \lambda_c\ \bar{\psi} := \boldsymbol{S} \circ \hat{\psi},\ \bar{\Pi} := \boldsymbol{S} \circ \Pi^\top \circ (1./\boldsymbol{S}),\ \bar{\omega} := \boldsymbol{S} \circ \omega$. With this notation, we have: $\boldsymbol{S} \circ \tilde{\lambda}_c := \boldsymbol{\Pi}(\boldsymbol{S} \circ \boldsymbol{\lambda}_c) = \Pi\bar{\boldsymbol{\lambda}}_c$.

These definitions imply that (138)–(142) become:

$$\begin{aligned}\bar{\boldsymbol{\psi}} &= \bar{\boldsymbol{\omega}} \circ \boldsymbol{\xi}^{u,0} \circ u'(\boldsymbol{c}) - \mu\boldsymbol{S} \quad (145)\\ &\quad - \left(\bar{\boldsymbol{\lambda}}_c \circ \boldsymbol{\xi}^{u,E} - (1+r)\boldsymbol{\Pi}\bar{\boldsymbol{\lambda}}_c \circ \boldsymbol{\xi}^{u,E} - w\bar{\boldsymbol{\lambda}}_l \circ \boldsymbol{y} \circ \boldsymbol{l} \circ \tilde{\boldsymbol{\xi}}^{u,1}\right) \circ u''(\boldsymbol{c}),\end{aligned}$$

$$\boldsymbol{P}\bar{\boldsymbol{\psi}} = \beta(1+r)\boldsymbol{P}\bar{\boldsymbol{\Pi}}\bar{\boldsymbol{\psi}}, \quad (146)$$

$$(\boldsymbol{I}-\boldsymbol{P})\bar{\boldsymbol{\lambda}}_c = 0, \tag{147}$$

$$\bar{\boldsymbol{\psi}} = \bar{\boldsymbol{\omega}} \circ \tilde{\boldsymbol{\xi}}^{v,0} \circ v'(\boldsymbol{l}) + \bar{\boldsymbol{\lambda}}_l \circ \tilde{\boldsymbol{\xi}}^{v,1} \circ v''(\boldsymbol{l}) - \mu F_L \boldsymbol{S}/w, \tag{148}$$

$$\tilde{\boldsymbol{a}}^\top \bar{\boldsymbol{\psi}} = -\left(\boldsymbol{\xi}^{u,E} \circ u'(\boldsymbol{c})\right)^\top \Pi \bar{\boldsymbol{\lambda}}_c, \tag{149}$$

$$(\boldsymbol{y}\circ \boldsymbol{l})^\top \bar{\boldsymbol{\psi}} = -\left(\boldsymbol{y}\circ \boldsymbol{l} \circ \tilde{\boldsymbol{\xi}}^{u,1} \circ u'(\boldsymbol{c})\right)^\top \bar{\boldsymbol{\lambda}}_l, \tag{150}$$

$$\boldsymbol{1}^\top \bar{\boldsymbol{\psi}} = 0, \tag{151}$$

which is a system in $\bar{\boldsymbol{\psi}}, \bar{\boldsymbol{\omega}}, \bar{\boldsymbol{\lambda}}_c, \bar{\boldsymbol{\lambda}}_l, \mu$, which characterizes the allocation.

G.4.2 FOCs in matrix form

Equation (148) yields:

$$\bar{\boldsymbol{\lambda}}_l = \boldsymbol{M}_0 \bar{\boldsymbol{\omega}} + \boldsymbol{M}_1 \bar{\boldsymbol{\psi}} + \mu \boldsymbol{V}_0, \tag{152}$$

with $\boldsymbol{M}_0 = -\boldsymbol{M}_1 \boldsymbol{D}_{\tilde{\boldsymbol{\xi}}^{v,0}\circ v'(\boldsymbol{l})}$, $\boldsymbol{M}_1 = \boldsymbol{D}^{-1}_{\tilde{\boldsymbol{\xi}}^{v,1}\circ v''(\boldsymbol{l})}$, $\boldsymbol{V}_0 = F_L \boldsymbol{M}_1 \boldsymbol{S}./w$. Then, equation (**??**) implies:

$$\bar{\boldsymbol{\psi}} = \hat{\boldsymbol{M}}_0 \bar{\boldsymbol{\omega}} + \hat{\boldsymbol{M}}_1 \bar{\boldsymbol{\lambda}}_c + \hat{\boldsymbol{M}}_2 \bar{\boldsymbol{\lambda}}_l - \mu \boldsymbol{S}, \tag{153}$$

with: $\hat{\boldsymbol{M}}_0 = \boldsymbol{D}_{\boldsymbol{\xi}^{u,0}\circ u'(\boldsymbol{c})}$, $\hat{\boldsymbol{M}}_1 = -\boldsymbol{D}_{\boldsymbol{\xi}^{u,E}\circ u''(\boldsymbol{c})}(\boldsymbol{I} - (1+r)\boldsymbol{\Pi})$, $\hat{\boldsymbol{M}}_2 = w\boldsymbol{D}_{\boldsymbol{y}\circ\boldsymbol{l}\circ\tilde{\boldsymbol{\xi}}^{u,1}\circ u''(\boldsymbol{c})}$. So using (152) and (153), we obtain:

$$\bar{\boldsymbol{\psi}} = \boldsymbol{M}_3 \bar{\boldsymbol{\omega}} + \boldsymbol{M}_4 \bar{\boldsymbol{\lambda}}_c + \mu \boldsymbol{V}_1, \tag{154}$$

with: $\boldsymbol{M}_2 = \boldsymbol{I} - \hat{\boldsymbol{M}}_2 \boldsymbol{M}_1$, $\boldsymbol{M}_4 = \boldsymbol{M}_2^{-1} \hat{\boldsymbol{M}}_1$, $\boldsymbol{M}_3 = \boldsymbol{M}_2^{-1}(\hat{\boldsymbol{M}}_0 + \hat{\boldsymbol{M}}_2 \boldsymbol{M}_0)$, $\boldsymbol{V}_1 = \boldsymbol{M}_2^{-1}(\hat{\boldsymbol{M}}_2 \boldsymbol{V}_0 - \boldsymbol{S})$. Then, using (**??**), (147), and (154), we get:

$$\bar{\boldsymbol{\lambda}}_c = \boldsymbol{M}_5 \bar{\boldsymbol{\omega}} + \mu \boldsymbol{V}_2, \tag{155}$$

with $\tilde{\boldsymbol{R}}_5 = -((\boldsymbol{I}-\boldsymbol{P}) + \boldsymbol{P}(I - \beta(1+r)\bar{\boldsymbol{\Pi}})\boldsymbol{M}_4)^{-1}\boldsymbol{P}(I - \beta(1+r)\bar{\boldsymbol{\Pi}})$, $\boldsymbol{M}_5 = \tilde{\boldsymbol{R}}_5 \boldsymbol{M}_3$, $\boldsymbol{V}_2 = \tilde{\boldsymbol{R}}_5 \boldsymbol{V}_1$. We then use equation (149), which becomes with (154) and (155): $\mu = -\boldsymbol{L}_1 \bar{\omega}$, with: $C_1 = \tilde{\boldsymbol{a}}^\top(\boldsymbol{V}_1 + \boldsymbol{M}_4 \boldsymbol{V}_2) + (\boldsymbol{\xi}^{u,E} \circ u'(\boldsymbol{c}))^\top \boldsymbol{\Pi} \boldsymbol{V}_2$, $\boldsymbol{L}_1 = (\tilde{\boldsymbol{a}}^\top(\boldsymbol{M}_3 + \boldsymbol{M}_4 \boldsymbol{M}_5) + (\boldsymbol{\xi}^{u,E} \circ u'(\boldsymbol{c}))^\top \boldsymbol{\Pi} \boldsymbol{M}_5)/C_1$. We deduce from (152), (154), and (155):

$$\bar{\boldsymbol{\lambda}}_l = \hat{\boldsymbol{M}}_6 \bar{\boldsymbol{\omega}},\ \bar{\boldsymbol{\lambda}}_c = (\boldsymbol{M}_5 - \boldsymbol{V}_2 \boldsymbol{L}_1)\bar{\boldsymbol{\omega}},\ \text{and}\ \bar{\boldsymbol{\psi}} = \boldsymbol{M}_6 \bar{\boldsymbol{\omega}},`$$

with: $\boldsymbol{M}_6 = \boldsymbol{M}_3 + \boldsymbol{M}_4(\boldsymbol{M}_5 - \boldsymbol{V}_2 \boldsymbol{L}_1) - \boldsymbol{V}_1 \boldsymbol{L}_1$ and $\hat{\boldsymbol{M}}_6 = \boldsymbol{M}_0 + \boldsymbol{M}_1 \boldsymbol{M}_6 - \boldsymbol{V}_0 \boldsymbol{L}_1$.

The constraints (150) and (151) become:

$$\boldsymbol{L}_3 \bar{\boldsymbol{\omega}} = \boldsymbol{L}_4 \bar{\boldsymbol{\omega}} = 0, \tag{156}$$

with: $\boldsymbol{L}_3 = (\boldsymbol{y}\circ\boldsymbol{l})^\top \boldsymbol{M}_6 + (\boldsymbol{y}\circ\boldsymbol{l}\circ\tilde{\boldsymbol{\xi}}^{u,1} \circ u'(\boldsymbol{c}))^\top \hat{\boldsymbol{M}}_6$, $\boldsymbol{L}_4 = \boldsymbol{1}^\top \boldsymbol{M}_6$.

G.4.3 Constructing the social weights

We assume that there are K distinct social weights, ω^s, where K is the number of productivity levels. Define $\boldsymbol{M}_7$ as the $N_{tot} \times K$ matrix of elements in $\{0,1\}$. The element of row h in column y is 1 if the current productivity of history h is y. We thus have: $\bar{\boldsymbol{\omega}} = \boldsymbol{D_S M}_7 \boldsymbol{\omega}^s$. We define the social weights as the ones that are the closest to utilitarian weights, such that constraints (156) hold. Formally, they are given as a solution of the following minimization problem:

$$\min_{\omega} \left\|\boldsymbol{\omega}^s - \mathbf{1}_K\right\|^2,$$
$$\text{s.t. } \boldsymbol{L}_3 \boldsymbol{D_S M}_7 \boldsymbol{\omega}^s = \boldsymbol{L}_4 \boldsymbol{D_S M}_7 \boldsymbol{\omega}^s = 0.$$

Denoting by $2\mu_3$ and $2\mu_4$ the Lagrange multipliers on the two constraints, the FOCs imply:

$$\boldsymbol{\omega}^s = \mathbf{1}_K + \sum_{k=3}^{4} \mu_k (\boldsymbol{L}_k \boldsymbol{D_S M}_7)^\top, \tag{157}$$

which once substituted in constraints (156) yield: $\begin{bmatrix} \mu_3 \\ \mu_4 \end{bmatrix} = \boldsymbol{M}_8^{-1} \boldsymbol{V}_8$, with:

$$\boldsymbol{M}_8 = \begin{bmatrix} \boldsymbol{L}_3 \boldsymbol{D_S M}_7 \\ \boldsymbol{L}_4 \boldsymbol{D_S M}_7 \end{bmatrix} \begin{bmatrix} \boldsymbol{L}_3 \boldsymbol{D_S M}_7 \\ \boldsymbol{L}_4 \boldsymbol{D_S M}_7 \end{bmatrix}^\top \text{ and } \boldsymbol{V}_8 = - \begin{bmatrix} \boldsymbol{L}_3 \boldsymbol{D_S M}_7 \mathbf{1}_K \\ \boldsymbol{L}_4 \boldsymbol{D_S M}_7 \mathbf{1}_K \end{bmatrix}.$$

Finally, from (157), we deduce the following expression for the social weights:

$$\boldsymbol{\omega}^s = \mathbf{1}_K + \boldsymbol{M}_8^{-1} \boldsymbol{V}_8 \begin{bmatrix} \boldsymbol{L}_3 \boldsymbol{D_S M}_7 \\ \boldsymbol{L}_4 \boldsymbol{D_S M}_7 \end{bmatrix}^\top.$$

H Monetary shocks and inequality

In this section, we solve the model to study the effect of a contractionary monetary shock of one standard deviation on the dynamics of inequality using a standard Taylor rule. As shown in Section 4, the dynamics of price inflation are sensitive to the choice of the fiscal rule. To abstract from the effect of fiscal policy, we assume that there is no public spending need ($G = 0$), hence zero tax. The rest of the calibration follows Section F. Firms' profits are distributed to households and we set $\nu = 10$ as in Section E.3. Because firms profits are null at steady-state, the steady state is the same as in Section F. We check that the dynamics of inequality are in line with the empirical evidence of Coibion et al. (2017). The nominal gross and pre-tax interest rate $\tilde{R}_t^N$ is set according to the following simple Taylor rule: $\frac{\tilde{R}_t^N}{\tilde{R}^N} = \left(\frac{\Pi_t^P}{\Pi^P}\right)^{\phi_\Pi} \zeta_t^{\text{Taylor}}$, where: (i) $\log(\zeta_t^{\text{Taylor}}) = \rho^{\text{Taylor}} \log(\zeta_{t-1}^{\text{Taylor}}) + \varepsilon_t^{\text{Taylor}}$ is a persistent monetary policy shock with $\rho^{\text{Taylor}} = 0.5$; (ii) $\tilde{R}^N$ and Π^P are steady-state values, and (iii) the response of the nominal interest rate to inflation is equal to $\phi_\Pi = 1.5$. These are standard values in this literature (see Galí, 2015).

We report in Figure 9 the IRFs for the innovation to the Taylor rule, the rate of inflation, and output. The responses of aggregate quantities to this contractionary monetary policy shock are consistent with empirical evidence and the rest of the literature (Kaplan et al., 2018 among others). The real interest rate falls, depressing consumption and investment and causing inflation and output to fall.

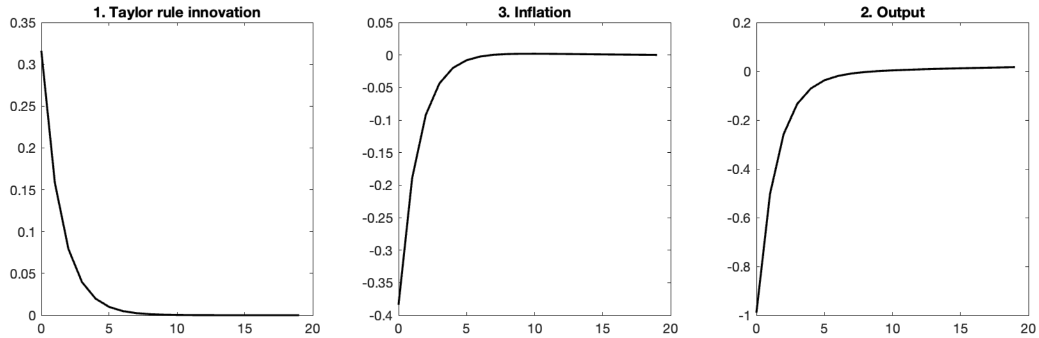

Figure 9: IRFs after a contractionary shock, in percentage deviation for all variables.

We also report in Figure 10 the responses of the Gini coefficients for income, consumption, and wealth corresponding to the same shock as in Figure 9. In line with the literature (see Gornemann et al., 2016 and Coibion et al., 2017), contractionary monetary policy increases these Gini coefficients and hence inequality.

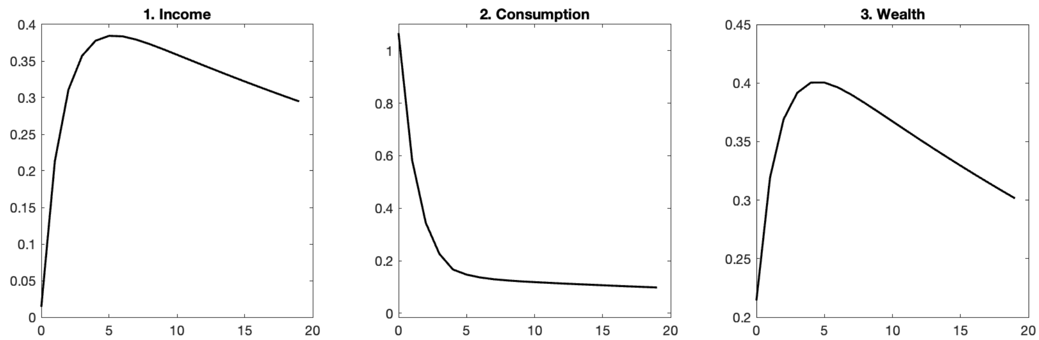

Figure 10: Gini coefficients of income, consumption, and wealth after the contractionary monetary shock of Figure 9. An increase by 1 indicates an increase of the Gini coefficient by 0.1 pp.

I The sticky-wage economy

I.1 The price Phillips curve

Following the New Keynesian sticky-wage literature, labor hours are supplied monopolistically by a continuum of unions of size 1. We denote by L_{kt} the labor supply of union k at date t and W_{kt} the related nominal wage. A competitive aggregator pools union-specific labor supplies together with a constant elasticity of substitution ε_W:

$$L_t = \left(\int_k L_{kt}^{\frac{\varepsilon_W - 1}{\varepsilon_W}} dk \right)^{\frac{\varepsilon_W}{\varepsilon_W - 1}}. \tag{158}$$

The objective of the competitive aggregator is to choose union labor supplies that minimize the total labor cost $\int_k W_{kt} L_{kt} dk$ subject to (158). We obtain:

$$L_{kt} = \left(\frac{W_{kt}}{W_t} \right)^{-\varepsilon_W} L_t, \tag{159}$$

where $W_t = \left(\int_k W_{kt}^{1-\varepsilon_W} dk \right)^{\frac{1}{1-\varepsilon_W}}$ is the nominal wage index.

The objective of each union k is to bargain the nominal wage W_{kt} that maximizes the intertemporal welfare of its members, while fulfilling the demand of (159). The nominal wage adjustment is assumed to imply a utility cost, equal to $\frac{\psi_W}{2}(W_{kt}/W_{kt-1} - 1)^2$. The objective of union k is thus: $\max_{(W_{ks})_s} \mathbb{E}_t \sum_{s=t}^{\infty} \beta^s \int_i \left(u(c_{i,s}) - v(l_{i,s}) - \frac{\psi_W}{2} \left(\frac{\hat{W}_{ks}}{\hat{W}_{ks-1}} - 1 \right)^2 \right) \ell(di)$, subject to (159) and where $c_{i,t}$ and $l_{i,t}$ are the consumption and labor supply of agent i. The FOC with respect to W_{kt} thus writes as:

$$\Pi_t^W (\Pi_t^W - 1) = \frac{W_{kt}}{\psi_W} \int_i \left(u'(c_{i,t}) \frac{\partial c_{i,t}}{\partial W_{kt}} - v'(l_{i,t}) \frac{\partial l_{i,t}}{\partial W_{kt}} \right) \ell(di) + \beta \mathbb{E}_t \Big[\Pi_{t+1}^W (\Pi_{t+1}^W + 1) \Big], \tag{160}$$

where the wage inflation rate is denoted by: $\Pi_t^W = W_{kt}/W_{kt-1}$. Agent i supplies l_{ikt} hours to every union k, such that $l_{it} = \int_k l_{ikt} dk$. Focusing on the symmetric equilibrium: $l_{ikt} = L_{kt}$, we thus deduce from (159):

$$W_{kt} \frac{\partial l_{i,t}}{\partial W_{kt}} = W_{kt} \frac{\partial \left(\int_k \left(\frac{W_{kt}}{W_t} \right)^{-\varepsilon_W} L_t dk \right)}{\partial \hat{W}_{kt}} = -\varepsilon_W L_{kt}. \tag{161}$$

The derivative of consumption $\frac{\partial c_{i,t}}{\partial W_{kt}}$ is equal to the derivative of the net total income, equal to $W_{kt} y_{i,t} l_{i,t} / P_t$. Formally:

$$W_{kt} \frac{\partial c_{i,t}}{\partial W_{kt}} = \left(l_{i,t} + W_{kt} \frac{\partial l_{i,t}}{\partial W_{kt}} \right) \frac{W_{kt} y_{i,t}}{P_t} = (1 - \varepsilon_W) W_{kt} L_{kt} \frac{y_{i,t}}{P_t}. \tag{162}$$

Because of the symmetric equilibrium assumption, we have $W_{kt} = W_t$ and $l_{it} = L_{kt} = L_t$.

Combining (160) with the partial derivatives (161) and (162), we deduce the following Phillips curve for wage inflation:

$$\Pi_t^W(\Pi_t^W - 1) = \frac{\varepsilon_W}{\psi_W}\underbrace{\left(v'(L_t) - \frac{\varepsilon_W - 1}{\varepsilon_W} w_t \int_i y_{i,t} u'(c_{i,t})\ell(di)\right)}_{\text{labor gap}} L_t + \beta\mathbb{E}_t\left[\Pi_{t+1}^W(\Pi_{t+1}^W - 1)\right], \quad (163)$$

where $w_t = W_t/P_t$ is the real pre-tax wage.

I.2 The rest of the environment

The rest of the economy is similar to the one described in Section 4 and we keep the same notation. Households solely invest in shares of mutual fund and equations (42) and (43) hold. Household's budget constraint is still expressed by equation (45), while their credit limit is given by equation (46). Their unique Euler equation is again expressed by (47). The governmental budget constraint is given by equation (48), while the financial market clearing condition is (49).

Monetary policy still consists in choosing the nominal interest rate $\tilde{R}_t^N$ on public debt (between t and $t+1$), affecting the gross inflation rate Π_t^P. But know, this price inflation path $(\Pi_t^P)_{t\geq 0}$ needs to be consistent with the paths of wage inflation $(\Pi_t^W)_{t\geq 0}$ and net wages $(w_t)_{t\geq 0}$. Indeed, with $\Pi_t^P = \frac{P_t}{P_{t-1}}$:

$$\Pi_t^W = \frac{W_t}{W_{t-1}} = \frac{w_t P_t}{w_{t-1}P_{t-1}} = \frac{w_t}{w_{t-1}}\Pi_t^P. \quad (164)$$

The fiscal parameters are driven by the set of acyclical fiscal rules (50)–(52).

I.3 Ramsey program with sticky wages

The Ramsey program can be written as:

$$\max_{\left(w_t, r_t, \tilde{w}_t, \tilde{r}_t^K, \tilde{R}_t^N, T_t, K_t, L_t, \Pi_t^W, \Pi_t^P, (a_{i,t}, c_{i,t}, \nu_{i,t})_i\right)_{t\geq 0}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\int_i (u(c_t^i) - v(L_t))\ell(di) - \frac{\psi_W}{2}(\Pi_t^W - 1)^2\right], \quad (165)$$

$$\text{s.t. } \Pi_t^W(\Pi_t^W - 1) = \frac{\varepsilon_W}{\psi_W}\left(v'(L_t) - \frac{\varepsilon_W - 1}{\varepsilon_W} w_t \int_i y_{i,t} u'(c_{i,t})\ell(di)\right) L_t + \beta\mathbb{E}_t\left[\Pi_{t+1}^W(\Pi_{t+1}^W - 1)\right], \quad (166)$$

$$\Pi_t^P = \frac{w_{t-1}}{w_t}\Pi_t^W, \quad (167)$$

and subject to: the fiscal path $(\tau_t^K, \tau_t^L, B_t)_{t\geq 0}$ following the fiscal rules (50)–(52), the governmental budget constraint (48), the household budget constraint (45), the household credit constraint (46), the Euler equations on consumption (47), the market clearing condition (49), the labor market clearing condition (19), the fund no-profit condition (42), the no-arbitrage condition (43), and the post-tax rate definitions (9) and (44).

Compared to the Ramsey program of Section 4.3 in the sticky-price economy, there are two main differences. First, the price Phillips curve (7) is replaced by the wage Phillips curve (166). The maximization thus further includes the wage inflation Π_t^W. The price inflation is still included, as it affects the interest rate in (42). It is connected to wage inflation through (167), Second, all households supply the same number of working hours L_t. The FOCs on labor (18) and the labor market clearing condition (19) are thus not constraints any more.

Denoting by $\beta^t\gamma_t^W$ the Lagrange multiplier of the wage Phillips curve, the Lagrangian can be written as:

$$
\begin{aligned}
\mathcal{L} = & \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \int_i (u(c_t^i) - v(L_t))\ell(di) - \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \frac{\psi_W}{2}(\Pi_t^W - 1)^2 \\
& - \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \int_i \left(\lambda_t^i - (1+r_t)\lambda_{t-1}^i\right) u'(c_t^i)\ell(di) \\
& - \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t (\gamma_t^W - \gamma_{t-1}^W)\Pi_t^W(\Pi_t^W - 1) + \frac{\varepsilon_W}{\psi_W}\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \gamma_t^W \left(v'(L_t) - \frac{\varepsilon_W - 1}{\varepsilon_W} w_t \int_i y_{i,t} u'(c_{i,t})\ell(di)\right) L_t \\
& + \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \mu_t \left(B_t + Y_t - \delta(\int_i a_{i,t-1}\ell(di) - B_{t-1}) - G_t - B_{t-1} - r_t \int_i a_{i,t-1}\ell(di) - w_t L_t - T_t\right) \\
& + \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^{t-1} \Upsilon_{t-1}(\frac{\tilde{R}_{t-1}^N}{\Pi_t^P} - (1+\tilde{r}_t^K)) + \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \Lambda_t \left(w_{t-1}\Pi_t^W - w_t \Pi_t^P\right) \\
& + \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \Gamma_t (r_t \int_i a_{i,t-1}\ell(di) - (1-\tau_t^K)(\tilde{r}_t^K K_{t-1} + (\frac{\tilde{R}_{t-1}^N}{\Pi_t^P} - 1)B_{t-1}).
\end{aligned}
$$

The planner's instruments are: $a_{i,t}$ for individual variables and L_t, $\tilde{R}_t^N$, r_t, Π_t^W and Π_t^P for aggregate variables.

Derivative with respect to $\tilde{R}_t^N$.

$$
\Upsilon_t E_t \left[\frac{1}{\Pi_{t+1}^P}\right] = \beta B_t E_t \left[(1-\tau_{t+1}^K)\frac{\Gamma_{t+1}}{\Pi_{t+1}^P}\right].
$$

Derivative with respect to Π_t^P.

$$
\Lambda_t w_t + \left(\beta^{-1}\Upsilon_{t-1} - \Gamma_t(1-\tau_t^K)B_{t-1}\right)\frac{\tilde{R}_{t-1}^N}{(\Pi_t^P)^2} = 0.
$$

Derivative with respect to Π_t^w.

$$
\psi_W(\Pi_t^W - 1) = \Lambda_t w_{t-1} - (\gamma_t^W - \gamma_{t-1}^W)(2\Pi_t^W - 1).
$$

Derivative wrt r_t.

$$\int_i \left(\psi_t^{SW,i} a_{t-1}^i + \lambda_{t-1}^i u'(c_t^i)\right) \ell(di) + (\Gamma_t - \mu_t) \int_i a_{i,t-1} \ell(di) = 0,$$

where $\psi_t^{SW,i}$ is defined in equation (69).

Derivative wrt L_t.

$$\begin{aligned} v'(L_t) \int_i \omega_t^i \ell(di) =& (1-\alpha) w_t \int \psi_t^{SW,i} y_t^i \ell(di) - \alpha \frac{w_t}{L_t} \left(\beta E_t \left[\Lambda_{t+1} \Pi_{t+1}^W\right] - \Pi_t^P \Lambda_t\right) \\ &+ \frac{\varepsilon_W}{\psi_W} \gamma_t^W \left[v'(L_t) + L_t v''(L_t) - \frac{\varepsilon_W - 1}{\varepsilon_W} (1-\alpha) w_t \int_i y_{i,t} u'(c_{i,t}) \ell(di)\right] \\ &+ \mu_t \left(\tilde{w}_t - w_t(1-\alpha)\right) - \frac{\alpha \tilde{w}_t}{K_{t-1}} \left(\beta^{-1} \Upsilon_{t-1} + (1-\tau_t^K) \Gamma_t K_{t-1}\right). \end{aligned}$$

Derivative wrt a_t^i.

$$\begin{aligned} \psi_t^{SW,i} =& \beta E_t \left[\psi_{t+1}^{SW,i} (1 + r_{t+1})\right] + \beta \alpha E_t \left[\frac{w_{t+1} L_{t+1}}{K_t} \int_i \psi_{t+1}^{SW,i} y_{t+1}^i \ell(di)\right] \\ &- \beta \frac{\varepsilon_W - 1}{\psi_W} \alpha E_t \left[\frac{w_{t+1} L_{t+1}}{K_t} \gamma_{t+1}^W \int_i y_{i,t+1} u'(c_{i,t+1}) \ell(di)\right] \\ &+ \beta E_t \left[\mu_{t+1} \left(\alpha \frac{Y_{t+1}}{K_t} - (\delta + r_{t+1}) - \alpha \frac{w_{t+1}}{K_t} L_{t+1}\right)\right] \\ &+ \beta \frac{(1-\alpha)}{K_t} E_t \left[(\tilde{r}_{t+1}^K + \delta) \left(\beta^{-1} \Upsilon_t \Gamma_{t+1} + \Gamma_{t+1} (1 - \tau_{t+1}^K) K_t\right)\right] + \beta E_t \left[\Gamma_{t+1} \left(r_{t+1} - (1 - \tau_{t+1}^K) \tilde{r}_{t+1}^K\right)\right] \\ &+ \beta E_t \left[\alpha \frac{w_{t+1}}{K_t} \left(\beta \Lambda_{t+2} \Pi_{t+2}^W - \Lambda_{t+1} \Pi_{t+1}^P\right)\right]. \end{aligned}$$

Derivative wrt T_t.

$$\mu_t = \int_j \psi_t^{SW,j} \ell(dj). \tag{168}$$

I.4 The implementation result

The property of the allocation where $\Pi_t^W = 1$ are easy to derive from the program of the planner. First, the post-tax real wage must satisfy (from equation (166)): $w_t \frac{\varepsilon_W - 1}{\varepsilon_W} \int_i y_{i,t} u'(c_{i,t}) \ell(di) = v'(L_t)$, and the price inflation is linked to the real wage dynamics $\Pi_t^P = w_{t-1}/w_t$, from equation (167). Using these expression in the program (62), one finds that the dynamics of the economy is given by the following equations:

$$G_t + B_{t-1} + r_t (B_{t-1} + K_{t-1}) + w_t L_t + T_t = B_t + Y_t - \delta K_{t-1}, \tag{169}$$

$$\text{for all } i \in \mathcal{I}\text{: } a_t^i + c_t^i = (1 + r_t) a_{t-1}^i + w_t y_t^i L_t + T_t, \tag{170}$$

$$a_t^i \geq -\bar{a}, \ \nu_t^i (a_t^i + \bar{a}) = 0, \ \nu_t^i \geq 0, \tag{171}$$

$$u'(c_t^i) = \beta \mathbb{E}_t \left[(1 + r_{t+1}) u'(c_{t+1}^i) \right] + \nu_t^i, \tag{172}$$

$$v'(L_t) = \frac{\varepsilon_W - 1}{\varepsilon_W} w_t \int_i y_{i,t} u'(c_{i,t}) \ell(di), \tag{173}$$

$$K_t + B_t = \int_i a_t^i \ell(di), \tag{174}$$

$$w_t = (1 - \tau_t^L) \tilde{w}_t, \tag{175}$$

$$r_t A_{t-1} = (1 - \tau_t^K) \left(\tilde{r}_t^K K_{t-1} + (\tilde{R}_{t-1}^N \frac{w_t}{w_{t-1}} - 1) B_{t-1} \right), \tag{176}$$

$$\tilde{R}_t^N \mathbb{E}_t \left[\frac{w_{t+1}}{w_t} \right] = \mathbb{E}_t \left[1 + \tilde{r}_{t+1}^K \right] / \mathbb{E}_t \left[\frac{w_{t+1}}{w_t} \right], \tag{177}$$

$$\tilde{r}_t^K + \delta = \alpha K_{t-1}^{\alpha-1} L_t^{1-\alpha}, \tag{178}$$

$$\tilde{w}_t = (1 - \alpha) K_{t-1}^{\alpha} L_t^{-\alpha}, \tag{179}$$

$$Y_t = Z_t K_{t-1}^{\alpha} L_t^{1-\alpha}, \tag{180}$$

together with the initial conditions a_{-1}^i, B_{-1} for all agents i, and with the fiscal rules (50)–(52). In this economy, the nominal interest rate is determined by the expectations of real variables (177) to implement $\Pi_t^W = 1$.

I.5 Simulating the sticky-wage economy

In this section we provide the truncated and matrix representations that allow us to simulate to sticky-wage economy (Sections I.5.1 and I.5.2). We also report some simulation results that complement those of the main text (Section I.5.3).

I.5.1 The truncated model

Similarly to Section D, and in particular to Section D.4, we derive the FOCs of the Ramsey program in the truncated model. The FOCs with respect to $\tilde{R}_t^N$, Π_t^P, Π_t^W, T_t are unchanged compared to the individual case. We define the truncated version of $\psi_t^{SW,i}$ of equation (69), that we still denote $\psi_{h,t}$ for the sake of simplicity as follows:

$$\psi_{h,t} = \xi_h^{u,0} u'(c_{h,t}) - (\lambda_{h,t} - (1 + r_t) \tilde{\lambda}_{h,t-1}) \xi_h^{u,E} u''(c_{h,t}) - \frac{\varepsilon_W - 1}{\psi_W} \gamma_t^W w_t L_t y_{h,t} \xi_h^{u,1} u''(c_{h,t}). \tag{181}$$

Derivative wrt r_t.

$$0 = \sum_h S_{h,t}\left(\psi_{h,t}\tilde{a}_{h,t} + \tilde{\lambda}_{h,t}\xi_h^{u,E}u'(c_{h,t})\right) + (\Gamma_t - \mu_t)A_{t-1}. \tag{182}$$

Derivative wrt L_t.

$$\begin{aligned} v'(L_t)\sum_h S_{h,t}\omega_{h,t} &= \mu_t(\tilde{w}_t - (1-\alpha)w_t) + \frac{\varepsilon_W}{\psi_W}\gamma_t^W(v'(L_t) + v''(L_t)L_t) \\ &\quad + (1-\alpha)w_t\sum_h\left(S_{h,t}y_{h,t}\psi_{h,t} - \frac{\varepsilon_W - 1}{\psi_W}\gamma_t^W S_{h,t}y_{h,t}u'(c_{h,t})\right)\Bigg) \\ &\quad - \alpha\tilde{w}_t\left(\beta^{-1}\frac{\Upsilon_{t-1}}{K_{t-1}} + \Gamma_t(1-\tau_t^K)\right) - \alpha\frac{w_t}{L_t}\left(\beta\mathbb{E}_t[\Lambda_{t+1}\Pi_{t+1}^W] - \Lambda_t\Pi_t^P\right). \end{aligned} \tag{183}$$

Derivative wrt $a_{h,t}$. For unconstrained truncated histories (i.e., $\nu_h = 0$):

$$\begin{aligned} \psi_{h,t} &= \beta\mathbb{E}_t\left[(1+r_{t+1})\sum_{\tilde{h}}\Pi_{h\tilde{h}}\psi_{\tilde{h},t+1}\right] \\ &\quad + \beta\alpha\mathbb{E}_t\frac{w_{t+1}L_{t+1}}{K_t}\sum_{\hat{h}}\left(S_{\hat{h},t+1}y_{\hat{h},t+1}\psi_{\hat{h},t+1} - \gamma_{t+1}^W\frac{\varepsilon_W - 1}{\psi_W}S_{\hat{h},t+1}y_{\hat{h},t+1}u'(c_{\hat{h},t+1})\right) \\ &\quad + \beta\mathbb{E}_t\left[\Gamma_{t+1}(r_{t+1} - (1-\tau_{t+1}^K)(\tilde{r}_{t+1}^K - (1-\alpha)(\tilde{r}_{t+1}^K + \delta)))\right] \\ &\quad + \beta\mathbb{E}_t\left[\frac{\mu_{t+1}}{K_t}(\alpha Y_{t+1} - K_t(\delta + r_{t+1}) - \alpha w_{t+1}L_{t+1})\right] \\ &\quad + \beta\alpha\mathbb{E}_t\left[\frac{w_{t+1}}{K_t}(-\Lambda_{t+1}\Pi_{t+1}^P + \beta\Lambda_{t+2}\Pi_{t+2}^W)\right] + \frac{(1-\alpha)}{K_t}\Upsilon_t\mathbb{E}_t[\tilde{r}_{t+1}^K + \delta]. \end{aligned} \tag{184}$$

For constrained agents, we have $a_{t,h} + \bar{a} = 0$ and $\lambda_{t,h} = 0$.

I.5.2 Matrix representation of the steady-state allocation

At the steady state, we have $Z = 1$, and the FOCs with respect to $\tilde{R}_t^N$, Π_t^P and Π_t^W at steady-state imply that $\Pi^W = \Pi^P = 1$ and $\Lambda = 0$. We use the same the notation as in Section D.5. We still have $\boldsymbol{S} = \boldsymbol{\Pi}^\top\boldsymbol{S}$ and $\boldsymbol{S}\circ\tilde{\boldsymbol{\lambda}}_c = \boldsymbol{\Pi}^\top(\boldsymbol{S}\circ\boldsymbol{\lambda}_c)$. The budget constraint implies: $\boldsymbol{S}\circ\boldsymbol{c} + \boldsymbol{S}\circ\boldsymbol{a} = (1+r)\boldsymbol{\Pi}^\top(\boldsymbol{S}\circ\boldsymbol{a}) + \boldsymbol{S}\circ\boldsymbol{W}\circ L$, while Euler equations still imply that we are looking for a vector $\boldsymbol{\xi}^u$ such that: $\boldsymbol{\xi}^u\circ u'(\boldsymbol{c}) = \beta(1+r)\boldsymbol{\Pi}\left(\boldsymbol{\xi}^u\circ u'(\boldsymbol{c})\right) + \boldsymbol{\nu}$. The FOC (182) with respect to r_t implies that we can express Γ as:

$$\Gamma = \mu - \frac{1}{A}\left[\tilde{\boldsymbol{a}}^\top\bar{\boldsymbol{\psi}} + (\boldsymbol{\xi}^{u,E}\circ u'(\boldsymbol{c}_h))^\top\left(\boldsymbol{\Pi}^\top(\boldsymbol{S}\circ\boldsymbol{\lambda})\right)\right]. \tag{185}$$

The definition (181) of $\psi_{h,t}$ implies that the vector $\bar{\psi} = S \circ \psi$ can be expressed as:

$$\begin{aligned} \bar{\boldsymbol{\psi}} &= \boldsymbol{M}_2 \bar{\boldsymbol{\lambda}} + \boldsymbol{L}_2 + \gamma^W \tilde{\boldsymbol{L}}_2, \\ \text{with: } \boldsymbol{M}_2 &= -\boldsymbol{D}_{\boldsymbol{\xi}^{u,E} \circ u''(\boldsymbol{c})} \left(\boldsymbol{I} - (1+r) \boldsymbol{\Pi}^\top \right), \\ \boldsymbol{L}_2 &= \boldsymbol{S} \circ \boldsymbol{\xi}^{u,0} \circ u'(\boldsymbol{c}), \\ \tilde{\boldsymbol{L}}_2 &= -\frac{\varepsilon_W - 1}{\psi_W} wL \boldsymbol{S} \circ \boldsymbol{y} \circ \boldsymbol{\xi}^{u,1} \circ u''(\boldsymbol{c}). \end{aligned} \tag{186}$$

The FOC (184) with respect to $a_{h,t}$ becomes:

$$\begin{aligned} \boldsymbol{P} \bar{\boldsymbol{\psi}} &= \beta(1+r) \boldsymbol{P} \boldsymbol{\Pi}^\psi \bar{\boldsymbol{\psi}} + \beta\alpha \frac{wL}{K} \boldsymbol{P} \boldsymbol{S} (\boldsymbol{y})^\top \bar{\boldsymbol{\psi}} - \beta\alpha \frac{\varepsilon_W - 1}{\psi_W} \frac{wL}{K} \gamma^W \boldsymbol{P} \boldsymbol{S} \boldsymbol{y}^\top (\boldsymbol{S} \circ u'(\boldsymbol{c})) \\ &+ \frac{\beta}{K} \boldsymbol{P} \boldsymbol{S} \mu \left(\alpha \left(Y - wL \right) - (\delta + r) K \right) + \beta \Gamma \boldsymbol{P} \boldsymbol{S} (1 - \tau^K)(1-\alpha)(\tilde{r}^K + \delta) \frac{A}{K}. \end{aligned}$$

Using (185) to substitute Γ, we obtain:

$$\begin{aligned} \boldsymbol{P} \tilde{\boldsymbol{L}}_1 \bar{\boldsymbol{\psi}} &= \boldsymbol{P} \boldsymbol{M}_1 \bar{\boldsymbol{\lambda}} + \boldsymbol{P} \boldsymbol{L}_1 \mu + \boldsymbol{P} \boldsymbol{N}_1 \gamma^W, \\ \text{with: } \tilde{\boldsymbol{L}}_1 &= \boldsymbol{I} - \beta(1+r) \boldsymbol{\Pi}^\psi - \alpha \frac{\beta}{K} wL \boldsymbol{S} (\boldsymbol{y})^\top + \frac{\beta}{K} (1-\alpha)(r + \delta(1-\tau^K)) \boldsymbol{S} \tilde{\boldsymbol{a}}^\top, \\ \boldsymbol{L}_1 &= \frac{\beta}{K} \left(\alpha \left(Y - wL \right) - K \left(r + \delta \right) + (1-\tau^K)(1-\alpha)(\tilde{r}^K + \delta) A \right) \boldsymbol{S}, \\ \boldsymbol{M}_1 &= -\beta \frac{1}{K} (1-\alpha) \left(r + \delta(1-\tau^K) \right) \boldsymbol{S} (\boldsymbol{\xi}^{u,E} \circ u'(\boldsymbol{c}))^\top \boldsymbol{\Pi}^\top, \\ \boldsymbol{N}_1 &= -\beta\alpha \frac{\varepsilon_W - 1}{\psi_W} \frac{wL}{K} \boldsymbol{S} \boldsymbol{y}^\top (\boldsymbol{S} \circ u'(\boldsymbol{c})). \end{aligned} \tag{187}$$

Combining (187) with the definition (186) of $\bar{\boldsymbol{\psi}}$ and the fact that for unconstrained agents, we have: $(\boldsymbol{I} - \boldsymbol{P}) \bar{\boldsymbol{\lambda}} = 0$, we obtain the following expression for $\bar{\boldsymbol{\lambda}}$:

$$\bar{\boldsymbol{\lambda}} = \tilde{\boldsymbol{L}}_3 \mu + \boldsymbol{L}_3 + \hat{\boldsymbol{L}}_3 \gamma^W, \tag{188}$$

with $\boldsymbol{M}_3 = \boldsymbol{I} - \boldsymbol{P} + \boldsymbol{P}(\tilde{\boldsymbol{L}}_1 \boldsymbol{M}_2 - \boldsymbol{M}_1)$, $\tilde{\boldsymbol{L}}_3 = \boldsymbol{M}_3^{-1} \boldsymbol{P} \boldsymbol{L}_1$, $\boldsymbol{L}_3 = -\boldsymbol{M}_3^{-1} \boldsymbol{P} \tilde{\boldsymbol{L}}_1 \boldsymbol{L}_2$, $\hat{\boldsymbol{L}}_3 = \boldsymbol{M}_3^{-1} \boldsymbol{P}(\boldsymbol{N}_1 - \tilde{\boldsymbol{L}}_1 \tilde{\boldsymbol{L}}_2)$.

Plugging the expression (188) into the definition (186) of $\psi_{h,t}$, we obtain:

$$\bar{\boldsymbol{\psi}} = \boldsymbol{L}_4 + \tilde{\boldsymbol{L}}_4 \mu + \hat{\boldsymbol{L}}_4 \gamma^W, \tag{189}$$

with $\boldsymbol{L}_4 = \boldsymbol{M}_2 \boldsymbol{L}_3 + \boldsymbol{L}_2$, $\tilde{\boldsymbol{L}}_4 = \boldsymbol{M}_2 \tilde{\boldsymbol{L}}_3$, $\hat{\boldsymbol{L}}_4 = \boldsymbol{M}_2 \hat{\boldsymbol{L}}_3 + \tilde{\boldsymbol{L}}_2$.

The FOC (183) with respect to L_t combined with the expression (185) yields:

$$v'(L) = \boldsymbol{L}_5^\top \bar{\boldsymbol{\psi}} + \hat{C}_5 \gamma^W + \tilde{C}_5 \mu + \tilde{\boldsymbol{L}}_5^\top \bar{\boldsymbol{\lambda}}, \tag{190}$$

with $\boldsymbol{L}_5^\top = (1-\alpha) w \boldsymbol{y}^\top + (1-\alpha)(1-\tau^K) \frac{\tilde{r}^K + \delta}{L} \tilde{\boldsymbol{a}}^\top$, $\hat{C}_5 = \frac{\varepsilon_W}{\psi_W} (v'(L) + v''(L) L - (1-\alpha) \frac{\varepsilon_W - 1}{\varepsilon_W} w \boldsymbol{y}^\top (\boldsymbol{S} \circ$

$u'(\boldsymbol{c}))$, $\tilde{C}_5 = (1-\alpha)\frac{1}{L}(Y - A(1-\tau^K)(\tilde{r}^K+\delta) - wL)$, $\tilde{\boldsymbol{L}}_5^\top = (1-\alpha)(1-\tau^K)\frac{\tilde{r}^K+\delta}{L}(\boldsymbol{\xi}^{u,E} \circ u'(\boldsymbol{c}))^\top \boldsymbol{\Pi}^\top$.

Substituting the expressions (186) and (188) of $\bar{\boldsymbol{\psi}}$ and $\bar{\boldsymbol{\lambda}}$, the previous equation becomes:

$$0 = C_6 + \tilde{C}_6\mu + \hat{C}_6\gamma^W, \tag{191}$$

with $C_6 = \boldsymbol{L}_5^\top \boldsymbol{L}_4 + \tilde{\boldsymbol{L}}_5^\top \boldsymbol{L}_3 - v'(L)$, $\tilde{C}_6 = \boldsymbol{L}_5^\top \tilde{\boldsymbol{L}}_4 + \tilde{\boldsymbol{L}}_5^\top \tilde{\boldsymbol{L}}_3 + \tilde{C}_5$, $\hat{C}_6 = \boldsymbol{L}_5^\top \hat{\boldsymbol{L}}_4 + \tilde{\boldsymbol{L}}_5^\top \hat{\boldsymbol{L}}_3 + \hat{C}_5$.

The FOC (168) with respect to T_t can be written at the steady as $\boldsymbol{1}_T^\top \bar{\boldsymbol{\psi}} = \mu$, which with the expression (186) of $\bar{\boldsymbol{\psi}}$ becomes:

$$0 = C_7 + \tilde{C}_7\mu + \hat{C}_7\gamma^W, \tag{192}$$

with $C_7 = \boldsymbol{1}_T^\top \boldsymbol{L}_4$, $\tilde{C}_7 = \boldsymbol{1}_T^\top \tilde{\boldsymbol{L}}_4 - 1$, $\hat{C}_7 = \boldsymbol{1}_T^\top \hat{\boldsymbol{L}}_4$.

Finally, combing (191) and (192) allows us to deduce $\gamma^W = -\frac{\tilde{C}_7 C_6 - \tilde{C}_6 C_7}{\tilde{C}_7 \hat{C}_6 - \tilde{C}_6 \hat{C}_7}$ and $\mu = -\frac{\hat{C}_7 C_6 - \hat{C}_6 C_7}{\hat{C}_6 \tilde{C}_7 - \hat{C}_7 \tilde{C}_6}$. We thus obtain the expressions of γ^W and μ as a function of the allocation. We can then deduce the expressions of Γ, $\bar{\boldsymbol{\lambda}}$, and $\bar{\boldsymbol{\psi}}$ using (185), (188), and (186). We then obtain L from equation (190), while Υ is given by: $\Upsilon = \beta(1-\tau^K)\Gamma B$.

I.5.3 Additional simulation results

We provide here some simulation results that complement those of Section 6.4. First, we report in Table 13 the steady-state distribution of wealth implied by the sticky-wage model. The distribution is very similar to the one implied by the sticky-price model (see Table 2). The conclusions are thus similar to the one of Section 5.1: the model does a good job in reproducing the data, but at the very top of the distribution.

	Data		Model
Wealth statistics	PSID, 06	SCF, 07	
Q1	−0.9	−0.2	0.0
Q2	0.8	1.2	0.3
Q3	4.4	4.6	5.4
Q4	13.0	11.9	18.7
Q5	82.7	82.5	76.6
Top 5%	36.5	36.4	34.5
Gini	0.77	0.78	0.73

Table 13: Wealth distribution in the data and in the sticky-wage model.

Second, we report the outcomes of the model using a cyclical fiscal rule. We use the same fiscal rule as in Section D.8 for the sticky-price economy. The rule is given in equations equations (50)–(51), with the same parameter values as in Section D.8: $(\sigma_1^L, \sigma_2^L, \sigma_1^K, \sigma_2^K, \sigma^T, \sigma^B) = (-0.6, 0.5, 0.6, -0.5, 8.5, 4.0)$. We conduct the same experiment as in the sticky-price economy.

We simulate the sticky-wage model after a negative TFP shock of one standard deviation. We plot the IRFs in Figure 11. We call Economy 4 the economy with the time-varying fiscal rule and optimal monetary policy.

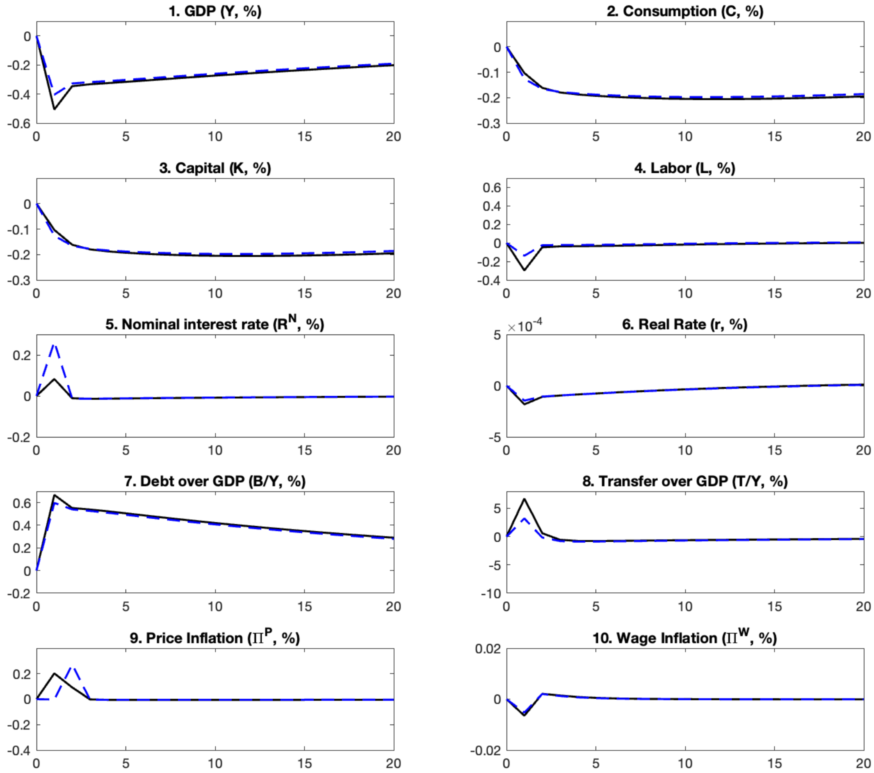

Figure 11: IRFs after a negative productivity shock of one standard deviation for relevant variables. The black line is the Economy 1 with the acyclical fiscal rule and optimal monetary policy. The blue dashed line is Economy 4, with time-varying fiscal rule and optimal monetary policy.

I.5.4 Role of the slope of the wage-Phillips curve

As for the price Phillips curve, the empirical literature offers quite a large range of estimates for the slope of the wage Phillips curve. We consider four different values of the slope: 1% (our benchmark), 3% (Auclert et al., 2024), 18%, and 35% (these two last values are correspond to two specifications estimated in Beraja et al., 2019). We then simulate the model after a negative TFP shock with these different slope values. More precisely, as in the price case, we keep the value of the elasticity ε_w unchanged to 21 and consider different values of the wage adjustment cost ψ_w implied by the various values of the slope – corresponding to ε_w/ψ_w in the Rotemberg model. The rest of the calibration is the same as the baseline one of Table 3. We report the quarterly wage and price inflation implied by the model simulations in Figure 12. Results confirm those of the main text. Wage inflation barely moves, since the maximal absolute movement is smaller than 0.04% (on a quarterly basis). Oppositely, price inflation moves by a sizable amount to allow the real wage to adjust and equalize the marginal productivity of labor..

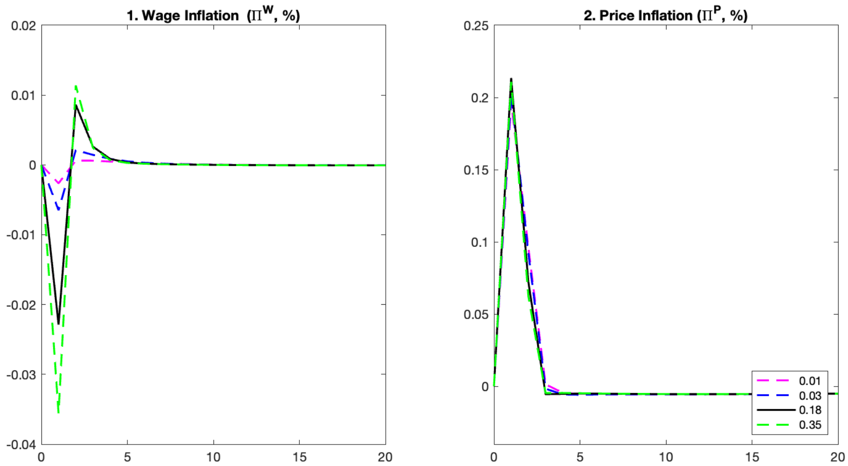

Figure 12: IRFs in percentage points after a negative TFP shock in the sticky-wage economy. Quarterly wage inflation (left-hand side) and price inflation (right-hand side) are given in percentage points. The different curves correspond to the different values of the slope of the Phillips curve, given in the legend.